PROGRESS OF SCIENCE IN INDIA

SECTION I

MATHEMATICS

[INCLUDING GEODESY AND STATISTICS]

(1939—1950)

EDITED BY

N. R. SEN, D.Sc., F.N.I.

National Institute of Sciences of India

New Delhi

Price: Rupees Four & thirty-seven Naye Paise

PROGRESS OF SCIENCE IN INDIA

SECTION I

MATHEMATICS

[INCLUDING GEODESY AND STATISTICS]

1939—1950

EDITED BY

N. R. SEN, D.Sc., F.N.I.

National Institute of Sciences of India
New Delhi

CONTENTS

PART I: MATHEMATICS 1940—1950

EDITOR: A. C. BANERJEE, M.Sc., M.A., F.R.A.S., F.N.I.

ALGEBRA

R. C. Bose (1942*a*, *b*; 1947; 1949) has contributed a number of papers to the theory of balanced incomplete block designs. R. Vaidyanathswami has written a paper on partially ordered sets (1944). V. K. Bala Chandram (1948, 1949) has studied ideals of the distribution lattice and prime ideals and the theory of last residue-classes. D. P. Banerji (1947) has written a note on the self inverse module. K. S. Banerji (1949*a*, *b*) worked on the construction of Hadamard matrices and also on the variance factors of weighing designs in-between two Hadamard matrices. Y. Bhalotra and S. Chowla's work deals with some theorems concerning quintics insoluble by radicals (1942).

K. N. Bhattacharya (1946) has discovered a new solution in symmetrical balanced incomplete block designs. He has also written a note on two-fold triple system (1943). S. Chakravarti (1941, 1944, 1947) has studied algebraical identities and recurrents in a number of papers. K. Chandrasekharan (1944) has discussed partially ordered sets and symbolic logic. A. C. Chaudhury (1948) has studied quasi groups and non-associative systems. S. Chowla (1945) has furthered the theory of construction of balanced incomplete block-designs. The work of N. N. Ghosh (1944, 1945) relates to a new reduction theorem of matrices and Hermitian matrix. Hansraj Gupta (1940) has studied a problem in combinations.

Harish Chandra (1945, 1947, 1950*a*) has discussed the Algebra of Dirac matrices, the Algebra of meson matrices, and the radical of a Lie algebra. He has also studied Lie algebra and the Tannaka duality theorem (1950*b*). Q. M. Hussain 1945*a*, *b*; 1946; 1948*a*, *b*) has devoted a number of papers to the theory of symmetrical incomplete block designs. S. M. Kerawala has, with A. R. Hanafi (1947*a*, *b*: 1948) prepared tables of monomial symmetrical functions of various weights. Kerawala (1946) has also written a note on symmetrical incomplete block designs. In other papers he has considered self conjugate latin squares of prime degree (1947; 1948). Kerawala (1947*a*, *b*) has also worked on the asymptotic number of three deep-latin rectangles. V. S. Krishnan (1945, 1947, 1948, 1950) has extended the various aspects of the theory of partially ordered sets in a number of papers. His work also deals with the problem of the last residue class in the distributive lattice (1942).

P. Keshava Menon (1942, 1948) has evaluated certain determinants and written a note on homogeneous cubic equation. A. A. Krishnaswami Ayyangar (1943) has developed a general theory of tactical configuration. K. Kishen (1947) has worked on fractional replication of the general symmetrical factorial designs (1947).

F. W. Levi (1943) has written a treatise on Modern Algebra. He has contributed two papers on the theory of pairs of inverse modulus. He has also studied the properties of a skew field of a given degree. B. S. M. Rao has written a number of papers on the algebra of elementary particles and some in collaboration with others (1945; 1946; 1947). In collaboration with B. S. Shastri he has studied the limits for the roots of a polynomial equation (1940). K. R. Nair (1943) has studied certain inequality relationship among the combinatorial parameters of incomplete block designs. H. K. Nandi (1946*a*, *b*) has considered non-isomorphic solutions of balanced incomplete block designs. In another paper he deals with the relation between certain types of tactical configuration (1945). C. Radhakrishna Rao (1947*a*) has discussed certain factorial experiments derivable from combinatorial arrangements of arrays. His other papers deal with general methods of analysis for incomplete block designs and a certain class of arrangements. (1947*b*, 1949).

Narsing Murti (1942) has studied a problem in combinations. A. Narsinga Rao (1948) has given a geometrical discussion of the homogeneous cubic equation. S. Pankajam (1944) has discussed group operation in certain distribution lattices. Amritasagar Puri (1943) has studied some problems in the theory of equations. S. S. Shrikhande (1950) has also worked on incomplete block designs. H. Sircar (1949) has written a paper on a certain system of equations (1949). C. N. Srinivasaenger (1947) in collaboration with others has considered Kemmer's identity in combinatorial functions. T. Vijaya Raghvan (1942) has written a note on symmetrical polynomial functions of zeroes of polynomials. M. Ziauddin (1940) has prepared tables of symmetric functions for statistical purposes.

Bibliography

Balachandram, V. K. (1948). Ideals of the distribution lattice. *J. Indian Math. Soc.* (N.S.), **12.**

——— (1949). Prime ideals and the theory of last—residue—classes. *Ibid.*, **13.**

Banerji, D. P. (1947). On the self-inverse module. *Math. student*, **15**, 17-18.

Banerji, K. S. (1949*a*). On the construction of Hadamard matrices. *Sci. & Cult.*, **14**, 434-435.

——— (1949*b*). On the variance factors of weighing designs in between two Hadamard matrices. *Bull. Cal. statist. Ass.*, **2**, 38-42.

Bhalotra, Y. and Chowla, S. (1942). Some theorems concerning quintics insoluble by radicals. *Math. Stud.*, **10**, 110-112.

Bhattacharya, K. N. (1943). A note on two-fold triple system. *Sankhyā*, **6**, 313-314.

——— (1946). A new solution in symmetrical balanced incomplete block designs. *Ibid.*, **7**, 423-424.

Bose, R. C. (1942*a*). A note on two series of balanced incomplete block designs. *Bull. Calcutta math. Soc.*, **34**, 129-130.

——— (1942*b*). A note on the resolvability of balanced incomplete designs. *Sankhyā*, **6**, 105-110.

——— (1942*c*). A note on two combinational problems having applications in the theory of designs of experiments. *Sci. & Cult.* **8**, 192-193.

——— (1947). On a resolvable series of balanced incomplete block designs. *Sankhyā*, **8**, 249-256.

——— (1949). A note on Fisher's inequality for balanced incomplete block designs. *Ann. math. Statist.*, **20**, 619-620.

Chakravarti, S. (1941). On a few recurrents. *J. Indian Math. Soc.* (N.S.), **5**, 18-26.

——— (1944). Some further algebraic identities. *Ibid.*, **8**, 115-119.

——— (1947). Some identities and recurrents. *Ibid.*, **11**, 89-94.

Chandrasekharan, K. (1944). Partially ordered sets and symbolic logic. *Math. Stud.*, **12**, 14-24.

Chaudhury, A. C. (1948). Quasi-groups and non-associative systems I. *Bull. Calcutta math. Soc.*, **40**, 183-194.

Chowla, S., and Ryser, H. J. (1950). Combinatorial problems. *Canad. J. Math.*, **2**, 93-99.

Chowla, S. (1945). A contribution to the theory of construction of balanced incomplete block designs. *Proc. Lahore phil. Soc.*, **7**, 3.

Ghosh, N. N. (1944). A note on Hermitian matrix. *Bull. Calcutta math. Soc.*, **36**, 87-90.

——— (1945). On a new reduction theorem of matrices. *Ibid.*, **37**, 33-36.

Gupta, Hansraj (1940). A problem in combinations. *Math. Stud.*, **8**, 131-132.

Harishchandra (1945). Algebra of the Dirac matrices. *Proc. Indian Acad. Sci.*, **22A**, 30-41.

——— (1947). On the algebra of meson-matrices. *Proc. Camb. phil. Soc.*, **43**, 414-421.

——— (1950*a*). On the radical of a Lie algebra. *Proc. Amer. math. Soc.*, **1**, 14-17.

——— (1950*b*). Lie algebra and the Tannaka duality theorem. *Ann. Math., Princeton*, **2**, 51, 209-230.

Hussain, Q. M. (1945*a*). Symmetrical incomplete block designs. *Bull. Calcutta math. Soc.*, **3**, 115-123.

——— (1945*b*). On the totality of the solutions for the symmetrical incomplete block designs. *Sankhyā*, **7**, 204-208.

——— (1946). Impossibility of symmetrical incomplete block designs. *Ibid.*, **7**, 317-322.

——— (1948*a*). Structure of some incomplete block designs. *Ibid.*, **8**, 381-383.

——— (1948*b*). Alternative proof of the impossibility of the symmetrical design. *Ibid.*, **8**, 384.

Kar, S. C. (1948). Zur Analyse der Lörentzgroupe in algebraischer Darstellung. *Bull. Calcutta math. Soc.*, **40**, 163-172.

Kerawala, S. M. (1946). Note on symmetrical incomplete block designs. *Ibid.*, **38**, 190-192.

——— (1947*a*). The asymptotic number of 3 deep-latin rectangles. *Ibid.*, **39**, 71-72.

——— (1947*b*). Asymptotic solution of the problem des manages. *Ibid.*, **39**, 82-84.

——— (1947*c*). A note on self-conjugate latin squares of prime degree. *Math. Stud.*, **15**,

——— (1948). A note on self-conjugate latin squares of prime degree. *Ibid.*, **16**,

Kerawala, S. M. and Hanafi, A. R. (1941*a*). Table of monomial symmetric functions of weight 9. *Proc. nat. Acad. Sci., India*, A, **11**, 51-55.
——— (1941*b*). A table of symmetric functions of weight 10. *Ibid.*, **A**, **11**, 56-63.
——— (1948). Table of monomial symmetrical functions of weight 12 in terms of power sums. *Sankhyā*, **8**, 345-369.
Kashava Menon, P. (1942). The valuation of certain determinants *Math. Stud.*, **10**, 75-79.
——— (1949). On the homogeneous cubic equation. *Ibid.*, **16**, 20-24.
Kishen, K. (1947). On fractional replication of the general symmetrical factorial designs. *Curr. Sci.*, **16**, 138-139.
Krishnaswami Ayyangar, A. A. (1943). A general theory of tactical configurations. *J. Mysore Univ.*, **A**, **1**, 103-113.
Krishnan, V. S. (1942). The problem of the last residue class in the distributive lattice. *Proc. Indian Acad. Sci.*, **A**, **16**, 176-190.
——— (1945). The theory of homomorphism and congruences for partially ordered sets. *Ibid.*, **A**, **22**, 1-19.
——— (1947). Extensions of partially ordered sets. *J. Indian math. Soc.* (N.S.), **11**, 49-68.
——— (1948). Extensions of partially ordered sets II. Constructions. *Ibid.*, **12**, 89-106.
——— (1950*a*). L' extension d' une (L,) algebre à une (Σ^*) algebre. *C. R. Acad., Sci., Paris*, **230**, 1447-1448.
——— (1950b). L'extension d'une (L,) algebre à une (Σ^*) algebre. *Ibid.*, **230**, 1559-1561.
Levi, F. W. (1940). Remarks on Mr. V. Narasingha Murthi's paper on a problem of arrangements. *J. Indian math. Soc.*, **4**, 45-46.
——— (1942). Algebra, Vol. I, Univ. of Calcutta.
——— (1943). Modern Algebra. *Bull. Calcutta math. Soc.*, **35**, 1-6.
——— (1947*a*). Pairs of inverse modules in a skew field. *Bull. Ameri. math. Soc.*, **53**, 1177-1182.
——— (1947b). On a skew field of 9 given degree. *J. Indian math. Soc.* (N.S.), **11**, 85-88.
Madhava Rao, B. S. (1945). Pauli's identities in the Dirac algebra. *Proc. Indian Acad. Sci. A*, **22**, 408-422.
——— (1947). Generalized algebra of elementary particles. *Proc. Indian Acad. Sci.*, *A*, **26**, 221-233.
——— and Shastri, B .S. (1940). On the limits for the roots of a polynomial equation. *J. Mysore Univ.*, *B*, **1**, 5-8.
——— Thiruvenkatachar, V. R., and Venkatachaliengar, K. (1946). Algebra related to elementary particles of spin 3/2. *Proc. roy. Soc. London*, *A*, **187**, 385-397.
Nair, K. R. (1943). Certain inequality relationships among the combinational parameters of incomplete block designs. *Sankhyā*, **6**, 255-259.
Nandi, H. K. (1945). On the relation between certain types of tactical configurations. *Bull. Calcutta math. Soc.* **37**, 92-94.
——— (1946*a*). Enumeration of non-isomorphic solutions of balanced incomplete block designs. *Sankhyā*, **7**, 305-312.
——— (1946*b*). A further note on non-isomorphic solutions of incomplete block designs. *Ibid.*, **7**, 313-316.
Narsing Murti (1942). A problem in combinations. *Math Stud.*, **10**, 85-86.
Narsinga Rao, A. (1948). On the homogeneous cubic equation—Geometrical approach. *Ibid.*, **16**, 25-27.
Pankajam, S. (1944). Group operation in certain distributive lattices. *Ibid.*, **12**, 25-29.
Puri, Amritasagar (1943). On some problems in the theory of equations. *Ibid.*, **11**, 15-20.
——— (1947*a*). Factorial experiments derivable from conbinatorial arrangements of arrays. *Sup. J. Ry. Statist. Soc.*, **9**, 128-139.
Radhakrishna, Rao C. (1947*b*). General methods of analysis for incomplete block designs. *J. Amer. Statist., Ass.*, **42**, 541-561.
——— (1949). On a class of arrangements. *Proc. Edinb. math. Soc.*, (2) **8**; 119-125.
Shrikhande, S. S. (1950). The impossibility of certain symmetrical balanced incomplete block designs. *Ann. math. Statist.*, **21**, 106-111.
Sirkar, H. (1949). On a system of equations. *Proc. Edinb. math, Soc.*, **8**, 2, 101-105.
Srinivasaengar, C. N., Gupta, Hansraj, Chakravarty, S. C., Venkatachaliengar K., and Thiruvenkatachar, V. R. (1947). On Kemmer's identity in combinatory functions. *Math. Stud.*, **15**, 93-100.
Vaidyanathswami, R. (1944). Partially ordered sets. *Math. Stud.*, **12**, 1-6.
Vijayaraghavan, T. (1942). On symmetric polynomial functions of zeros. of polynomials. *Math. Stud.*, **10**, 113-114.
Ziauddin, M. (1940). Tables of symmetrical functions for statistical purposes. *Proc. nat. Acad. Sci., India*, A, **10**, 53-60.

ANALYSIS

D. P. Banerji has given the expansion of some functions in terms of Sonine's polynomials and Whittaker confluent hypergeometric functions (1940, 1941). K. Basu also has written a note on Sonine's polynomials (1943). Ganapathy Iyer, V has made investigations in singular functions (1944). P. K. Ghosh has considered certain convergent integrals and their application to Mathematical Physics (1947, 1948). K. S. K. Iyenger has discussed Frullani Integrals (1941, 1940). P. Kesava Menon has generalised the circular and hypergeometric functions (1940).

G. S. Mahajani has given a generalization of the expansions of $\log(1+x)$, $(1+x)^m$ and e^x (1947*a*, *b*). He has also written a note on Riemannian Integration (1941). C. T. Rajagopal has commented on the remainder in Taylor's theorem (1946). V. Ramaswami has discussed the continuity of convex functions (1943). In another paper he considered Lagrange's form of the general mean-value theorem (1948). S. N. Roy has considered a certain class of multiple integrals (1945).

Bibliography

Banerji, D. P. (1940). On the expansion of a function in a series of Sonine's polynomials. *Bull. Calcutta math. Soc.*, **32**, 61-64.

——— (1941). On some expansions containing Laguerre polynomials and their expressions in terms of Whittaker's confluent hypergeometric functions. *Phil. Mag.*, (7), **32**, 84.

Basu, K. (1943). A note on Sonine's polynomials II. *Bull. Calcutta math. Soc.*, **35**, 127-130.

Chakravarti, M. C. (1948). On the ratio of the mean deviation to the standard deviation. *Bull. Calcutta Statist. Ass.* **1**, (**4**), 187-190.

Ganapathy Iyer, V. (1944). On singular functions. *J. Indian math. Soc.*, (N.S), **8**, 94-108.

Ghosh, P. K. (1947). On (C,1)-convergent integrals and their application to mathematical physics. *Bull. Calcutta math. Soc.*, **39**, 19-29.

——— (1948). On (C, α)-convergent integrals and their application to mathematical physics. *Ibid.*, **40**, 1-7.

Iyenger, K. V. (1942). A deepening of the binomial inequality. *J. Mysore Univ.*, B, **3**, 135-138.

Iyengar, K.S.K. (1940). On Frullani integrals. *J. Indian math. Soc.* (N.S.), **4**, 145-150.

——— (1941). On Frullani integrals. *Proc. Camb. phil. Soc.*, **37**, 9-13.

Kesava Menon P. (1940). A generalization of the circular and hypergeometric functions. *Math. Stud.*, **8**, 112-117.

Mahajani, G. S. (1941). A note on Riemann integration. *J. Univ. Bombay* (N.S.), **10**, (3), 9-11.

——— (1947*a*). An interesting result in the logarithmic expansion. *Ibid.*, **15**, (5),A, 1-2.

——— (1947*b*). Generalisation of the expansions of log $(1+x)$, $(1+x)^m$and e^x. Ibid., (N.S.), **16**, (3), A, 1-10.

Nair, U. S. and Mahajani, G. S. (1945). Generalization of a certain integral. *Math. Stud.*, **13**, 55-56.

Rajagopal, C. T. (1946). On the remainder in Taylor's theorem. *Math. Stud.*, **14**, 71-73.

Ramaswami, V. (1943). On the continuity of convex functions. *J. Benares Hindu Univ.*, **7**, (1), 180-181.

——— (1948). On polynomials and Lagrange's form of the general mean value theorem. *Bull. Amer. math. Soc.*, **54**, 946-949.

Roy, S. N. (1945). On a certain class of multiple integrals. *Bull. Calcutta math. Soc.*, **37**, 69-76.

Sengupta, H. M. (1948). On continuous independent functions. *Quart. J. Math.*, **9**, No. 75, 129-132.

——— (1950). On rearrangements of series. *Proc. Amer. math. Soc.*, **1**, No. 1, 71-75.

Sharma, A. (1945). On the minimal interval of ξ in the second mean-value theorem. *Proc. Benares math. Soc.*, (N.S.), **7**, No. 2, 33-40.

Shastri, H. Srinivasa (1940). On the function defined by the relation $\phi_r(n) = n\phi'_{r-1}(n)$. *Math. Stud.*, **8**, 127-130.

Sircar, H. (1945). On the inequality satisfied by the derivative of order n of a function. *Bull. Calcutta math. Soc.*, **77**, 21-23.

ASTRONOMY

S. Chandrasekhar who is settled in America, has to his credit an admirable contribution to the science of Astronomy and Astrophysics. He has expounded the fundamental principles of stellar dynamics in a series of papers which have been collected in book form. Taking account of dynamical friction he has propounded a theory on the rate of escape of stars from clusters. He has made a detailed investigation into the statistics of the gravitational field arising from a random distribution of stars and the correlaion in the forces acting at two points separated by a finite distance. He has written a series of important papers on the Radiative Equilibrium of a stellar atmosphere. (1940*a*, *b* ; 1943*a*, *b*, *c* ; 1944*a*, *b*, *c*, *d*, *e* ; 1945*a*, *b* ; 1946*a*, *b*). He has also written a paper on Brownian motion, dynamical friction and stellar dynamics (1949).

A. C. Banerji has made a deep study of the adiabatic radial oscillations of a cepheid variable and having proved the instability of such oscillations, he has developed a new theory about the origin of the solar system (1942a). He delivered extension lectures on Galactic Dynamics in which he made critical and schloastic study of the recent advances (1942*b*). D. S. Kothari has written a paper on the source of energy in a white dwarf star (1940). Along with F. C. Auluck he has investigated the minimum radius for degenerate stellar masses (1947). N. R. Sen has studied the constitution of stellar models some of them based on Bethe's Law of energy generation (1942*a*, *b*). He has written a paper on the inversion of density gradient and convection in stellar bodies (1941). He has also written a note on pressure relations within fluid spheres in equilibrium (1944). In collaboration with U. R. Burman he has investigated into the internal constitution of stars of small masses according to Bethe's law of energy generation (1945).

U. R. Burman integrated the stellar equations for Bethe's law of energy generation (1942). In another paper he has studied the fitting of an adiabatic core to a radiative envelope in stellar models (1943). He has studied the problem of the Helium content of the stars of large masses (1947). He has also investigated the question of fitting of convective cores of different compositions to a given envelope solution of the stellar equations (1949). P. L. Bhatnagar has contributed a number of papers on the theory of Anharmonic pulsations and radial oscillations of a cepheid variable (1945*a*, *b* ; 1946*a*, *b*, *c*). Sunil Kumar Roy has studied polytropic gaseous stars rotating with variable angular velocity (1942 ; 1943 ; 1944*b*). He has also investigated the Anharmonic pulsations of the cepheid variables (1944*a*).

H. K. Sen has made a detailed study of the adiabatic radial oscillations of gas spheres under various laws of density variation (1942*a*, *b*, *c*, *d*, *e* ; 1943*a*, *b*, *c*). He has also written a paper on the radiative equilibrium of a spherical planetary nebulae (1949). Braj Basi Lal has studied the problem of the formation of the arms of the spiral nebula in resisting medium under various laws (1943*a*, *b*, *c*, *d*). In another paper he has discussed the theory of a spiral nebula (1942). N. L. Ghosh has studied the problem of relative equilibrium of fluid matter in rotation (1947, 1948, 1949). Chandrika Prasad has written a number of papers on the theory of radial oscillations and anharmonic oscillations of stellar models based on different laws of density variations (1947, 1949*a*, *b*). He has studied the stability of Maclaurin Spheroids rotating with constant angular velocity and has shown that the series of spheroids becomes unstable at the first point of bifurcation and remains so for the rest of the series (1950). G. Bandopadhyaya has studied slow homogeneous contraction of stars (1948).

Bibliography

Auluck, F. C. and Kothari, D. S. (1947). A note on the minimum radius for degenerate stellar masses. *Phil. Mag.*, (7), **38**, 368-370.

Bondyopadhyay, G. (1948). On slow homogeneous contraction of stars. *Proc. nat. Inst. Sci. India*, **14**, 29-43.

Banerji, A. C. (1942*a*). The instability of radial oscillations of a variable star and the origin of the solar system. *Proc. nat. Inst. Sci. India*, **8**, 173-197.

——— (1942*b*). Recent Advances in Galactic Dynamics. *Lucknow Univ. Stud.*, No. 15. Allahabad Law Journal Press, Allahabad, India.

Bhatnagar, P. L. (1945*a*). The theory of anharmonic pulsation for cepheids: Two-phase homogeneous model. *Proc. nat. Inst. Sci. India*, **11**, 13-20.

——— (1945*b*). Application of Rayleigh's principle to cepheid pulsations. *Ibid.*, **11**, 25-29.

——— (1946*a*). Anharmonic pulsation of a homogenous star, effect of the ratio of specific heats. *Ibid.*, **38**, 34-38.

——— (1946*b*). Radial oscillations of a rotating star. *Bull. Calcutta math. Soc.*, **38**, 93-95.

——— (1946*c*). Radial oscillations of a star. *Ibid.*, **38**, 168-176.

Burman, U. R. (1942). On integration of stellar equation for Bethe's law of energy generation. *Proc. nat. Inst. Sci. India*, **8**, 301-316.

——— (1943). On the fitting of an adiabatic core to a radiative envelope in stellar models. *Proc. nat. Inst. Sci. India*, **9**, 237-243.

——— (1947). On the helium content of stars of large masses. *Proc. nat. Inst. Sci. India*, **13**, 179-186.

——— (1949). On the possibility of fitting of convective cases of different compositions to a given envelope solution of the stellar equations. *Proc. nat. Inst. Sci. India* **15**, 269-275.

Chandrasekhar, S. (1940*a*). The fundamental principles of stellar dynamics. *Astronomical Papers dedicated to Elis Stromgren*, pp. 1-24. Einar Munksgaard. Copenhegan.

——— (1940*b*). The dynamics of stellar systems. IX-XIV. *Astrophys. J.*, **92**, 441-642.

——— (1942*a*). Principles of stellar Dynamics. *Univ. of Chicago Press*, Chicago, IPL.

——— (1942*b*). A note on the perturbation theory for distorted stellar configurations. *Astrophys. J.*, **96**, 151-154.

——— (1943*a*). Dynamical friction. I. General considerations: the coefficient. of dynamical friction. *Ibid.*, **97**, 255-262.

——— (1943*b*). Dynamical friction. II. The rate of escape of stars from clusters and the evidence for the operation of dynamical friction. *Ibid.*, **97**, 263-273.

——— (1943*c*). Dynamical friction. III. A more exact theory of the rate of escape of stars from clusters. *Ibid.*, **98**, 54-60.

——— (1943*d*). New methods in stellar dynamics. *Ann. N. Y. Acad. Sci.*, **45**, 133-161.

——— (1944*a*). The statistics of the gravitational field arising from a random distribution of stars. III. The correlations in the forces acting at two points separated by a finite distance. *Astrophys. J.*, **99**, 25-46.

——— (1944*b*). The statistics of the gravitational field arising from a random distribution of stars. IV. The Stochastic variation of the force acting on a star. *Ibid.*, **99**, 47-53.

——— (1944*c*). On the radiative equilibrium of a stellar atmosphere. *Ibid.*, **99**, 180-190.

——— (1944*d*). On the radiative equilibrium of a stellar atmosphere. II. *Ibid.*, **100**, 76-86.

——— (1944*e*). On the radiative equilibrium of a stellar atmosphere. III. *Ibid.*, **100**, 117-127.

——— (1945*a*). On the radiative equilibrium of a stellar atmosphere. V. *Astrophys.* **101**, 95-107.

——— (1945*b*). On the radiative equilibrium of a stellar atmosphere. VII. *Ibid.*, **101**, 328-347.

——— (1945*c*). On the radiative equilibrium of a stellar atmosphere. VIII. *Ibid.*, **101**, 348-355.

——— (1946*a*). On the radiative equilibrium of a stellar atmosphere. XII. *Astrophys. J.*, **104**,191-202.

——— (1946*b*). On the radiative equilibrium of a stellar atmosphere. X. *Astrophys. J.*, **103**, 351-370.

——— (1948). Stellar dynamics. *Monthly notes. Royal. Astr. Soc.*, **105**, 124-134.

——— (1949). Brownian motion, dynamical friction, and stellar dynamics. *Rev. mod. Phys.*, **21**, 383-388.

Ghosh, N. L. (1947). A note on the equilibrium of fluid matter in a steady differential rotation. *Bull. Calcutta math. Soc.*, **39**, 131-138.

——— (1948). A note on Hamy's theorem. *Ibid.*, **40**, 229-230.

——— (1949). Gases with a pressure density relation, impossibility of confocal density distributions. *Ibid.*, **41**, 183-186.

Kothari, D. S. (1940). The source of energy in a white dwarf star. *Sci. and Cult.*, **6**, 309-310.

Lal, Braj Basi (1942). On the theory of a spiral nebula. *Proc. nat. Acad. Sci. India*, A, **12**, 108-120.
——— (1943*a*). The arms of a spiral nebula in resisting medium I. *Ibid.*, A, **13**, 19-27.
——— (1943*b*). On the theory of spiral nebula II. *Ibid.*, A, **13**, 28-36.
——— (1943*c*). The arms of a spiral nebula in resisting medium II. *Ibid.*, **13**, 165-170.
——— (1943*d*). Formation on the arms of a spiral nebula. *Ibid.*, A, **13**, 179-183.
Moghe, D. N. (1941*a*). On the theory of a spiral nebula I. *Proc. nat. Inst. Sci. India*, **7**, 141-160.
——— (1941*b*). On the theory of a spiral nebula, II. *Ibid.*, **7**, 161-175.
Prasad, Chandrika (1944). Configurations of a rotating incompressible mass of fluid surrounded by an atmosphere of small density. *Proc. Benares math. Soc.* (N.S.) **6**, 35-48.
——— (1947). On a general theorem on rotating masses by Jeans. *Ibid.*, **9**, 33-35.
——— (1949*a*). Radial oscillations of a stellar model. *Mon. Not. R. astr. Soc.*, **109**, 103-107.
——— (1949*b*). Anharmonic pulsations of two particular models. *Astrophys., J.*, **110**, 375-381.
——— (1950). On the stability of Maclaurian Spheroids rotating with constant angular velocity. *Q. J. Math. Ox*(2) **1**, 117-121.
Roy, Sunil Kumar (1942). On polytropes rotating with variable angular velocity (in line). *Proc. nat. Acad. Sci. India*, A, **12**, 1-8.
——— (1943). A case of polytropic equilibrium configuration with variable angular velocity. *Bull. Calcutta math. Soc.*, **35**, 85-98.
(1944*a*). On the anharmonic pulsations in the pulsation theory of the cepheid variable. *Bull. Calcutta math. Soc.*, **36**, 109-121.
——— (1944*b*). Rotational distortion of gaseous stars. *Proc. Benares math. Soc.* (N.S.), **6**, 49-66.
Sen, H. K. (1942*a*). Polytropic gas spheres with variable index. *Proc. nat. Acad. Sci. India* A, **12**, 13-36.
——— (1942*b*). Radial oscillations of a variable star. *Ibid.*, A, **12**, 99-107.
——— (1942*c*). Adiabatic pulsation of the cepheid variables. *Ibid.*, A, **12**, 177-189.
——— (1942*d*). Uniform radial oscillations of a star. *Ibid.*, A, **12**, 236-244.
——— (1942*e*). Radial oscillations of a slowly rotating star. *Ibid.*, A, **12**, 245-257.
——— (1943*a*). Radial oscillations of the generalised Roche's model. *Ibid.*, A, **13**, 44-53.
——— (1943*b*). Rotating cepheid. *Ibid.*, A, **13**, 159-164.
——— (1943*c*). Large radial oscillations of a star. *Ibid.*, A, **13**, 184-194.
——— (1949). The radiative equilibrium of a spherical planetary nebula. *Astrophys. J.* **110**, 276-287.
Sen, N. R. (1941). On the inversion of density gradient and convection in stellar bodies. *Proc. nat. Inst. Sci. India*, **7**, 183-196.
——— (1942*a*). On stellar models based on Bethe's law of energy generation. *Ibid.*, **8**, 317-330.
——— (1942*b*). Contribution to the theory of stellar models. *Ibid.*, **8**, 339-360.
——— (1943). On a problem in heat conduction in a sphere. *Bull Calcutta math. Soc.*, **35**, 131-134.
——— (1944). Note on pressure relations within fluid spheres in equilibrium. *Ibid.*, **36**, 147-152.
Sen, N. R. and Burman, U. R. (1945). On the internal constitution of stars of small masses according to Bethe's Law of energy generation. *Astrophys. J.*, **102**, 208-215.

DIFFERENTIAL EQUATIONS

S. Chandrashekhar has solved some new differential equations in connection with Astro-Physical problems (1945, 1946, 1948). D. D. Kosambi has dealt with partial differential equations (1948). N. N. Ghosh has investigated some properties of complex operational matrices (1946). S. Minakshisundaram has contributed a series of papers to the study of non-linear Partial differential equations of the parabolic type and Fourier Ansatz (1939*a*, 1942*a*, *b*, 1943*a*). In a number of papers he has developed the theory of expansion of an arbitrary function in a series of eigenfunctions of boundary value problems (1941, 1942*b*, 1943*b*, *c*, *d*).

S. K. Mitra has solved the boundary value problem of the motion of two spheres in infinite fluid in terms of definite integrals (1944*a*, *b*). Shanti Ram Mukherji has given solutions of some differential equations arising in viscous hydrodynamical flow (1942). B. R. Seth has dealt with some vibrational problems (1940, 1941). V. R. Thiruvenkatachar has solved the equation of telegraphy by operational methods (1940). By using the method of Laplace transformation he has given a solution of some boundary problems (1941).

Bibliography

Bose, S. N., and Kar, S. C. (1941). The complete solution of the equation :

$$\nabla^2\phi - \frac{\partial^2\phi}{C^2\partial t^2} - K^2\phi = -4\pi\rho\,(xyzt).$$ *Proc. nat. Inst. Sci. India* **7,** 93-102.

Chandrasekhar, S. (1945). The radiative equilibrium of an expanding planetory nebula. I. Radiation pressure in Lyman -α. *Astrophys. J.*, **102,** 402-426.

——— (1946). A new type of boundary-value problem in hyperbolic equations. *Proc. Camb. phil. Soc.*, **42,** 250-260.

——— (1948). The softening of radiation by multiple Compton scattering. *Proc. roy. Soc.*, A, **192,** 508-518.

Ghosh, N. N. (1946). On some properties of complex operational matrices. *Bull. Calcutta math. Soc.*, **38,** 96-100.

Kosambi, D. D. (1948). System of partial differential equations of the second order. *Quart. J. Math.*, **19,** 204-219.

Minakshi Sundaram, S. (1941). On the expansion of an arbitrary function in a series of eigenfunctions of boundary value problems. II. *J. Indian math. Soc.* (N.S.), **5,** 103-108.

——— (1942*a*). Studies in Fourier Ansatz and parabolic equations. *J. Madras Univ.*, **14,** 73-142.

——— (1942*b*). Studies in Fourier Ansatz and parabolic equations. *J. Indian math. Soc.* (N.S.), **6,** 41-50.

——— (1942*c*). On expansion in eigenfunctions of boundary value problems. III. The summability problem. *Ibid.*, **6,** 153-167.

——— (1943*a*). On expansion in eigenfunctions of boundary value problems IV. *Ibid.*, **7,** 17-24.

——— (1943*b*). On expansion in eigenfunctions of boundary value problems V. *J. Indian math. Soc.* (N.S.), **7,** 89-95.

——— (1943*c*). Fourier Ansatz and non-linear parabolic equations. *Ibid.*, **7,** 129-142.

——— (1943*d*). On expansion in eigenfunctions of boundary value problems. VI. *Bull. Calcutta math. Soc.*, **35,** 81-84.

Mitra, Samarendra Kumar (1944*a*). A new method of solution of the boundary value problems of Laplaces equation relating to two spheres. I. *Ibid.*, **36,** 31-39.

——— (1944*b*). On the boundary value problems of Laplace's equation relating to two spheres. A new method of solution. *Sci. and Cult.* **9,** 397-398.

Mukherjee, Santi Ram (1942). Solutions of some differential equations arising in problems of varying viscosity in hydrodynamics. *Proc. nat. Acad. Sci. India* A, **12,** 46-65.

Nagabhushanam, K. (1942). Infinitesimal automorphisms of the action form. *Math. Stud.*, **10,** 64-67.

Nath Sharma, Prithvi (1946). On some vibrational problems. *Math. Stud.*, **14,** 63-64.

Sen, N. R. (1942). A note on meson-wave. *Bull. Calcutta math. Soc.*, **34**, 61-65.
Seth, B. R. (1940). Transverse vibrations of triangular membranes. *Proc. Indian Acad. Sci.*, A, **12**, 487-490.
——— (1941). On the gravest mode of some vibrating systems. *Ibid.*, A, **13**, 390-394.
Sircar, H. (1943). On harmonic function. *Bull. Calcutta math. Soc.*, **35**, 71-75.
Thiruvenkatachar, V. R. (1940). Operational solution of the equation of telegraphy. *J. Mysore Univ.*, B. **1**, 9-13.
——— (1941). Solution of boundary value problems by double Laplace transformations. *Ibid.*, B, **1**, 115-121.

THEORY OF ELASTICITY

N. N. Ghosh has developed a matrix method of analysing strain and stress in hyperspace (1942). S. Ghosh has devoted a number of papers to the investigations of plane strain and plane stress in aelotropic plates (1942, 1943, 1944). He has found a new function by theoretical method of solving the torsion problem for some boundaries and has discussed the torsion and flexure of beams whose cross-sections are bounded by specified contours (1947, 1948*a*, *b*). D. N. Mitra has considered the flexure problem for certain specified boundaries (1948, 1949*b*, *c*). He has also worked out the torsion and flexure of an isotropic elastic cylinder whose cross-section is a semi-cardioid (1949*a*).

Bibhutibhusan Sen has written a paper on the stresses in an infinite strip due to an isolated couple acting at a point inside it (1942). In another paper he has investigated the problem of stresses due to forces and couples acting in the interior of an infinite elastic slab placed on a rigid foundation (1943*a*). He has examined critically the uniqueness theorem for problems of thin plates bent by normal pressures (1943*b*). He has studied boundary value problems of circular discs under body forces (1944*a*). He has devoted a number of papers to the investigation of stresses in elastic discs of a variety of shapes (1945*a*, *b*, *c* ; 1946*a*, *b* ; 1947 ; 1948.

A. M. Sengupta has written two papers on the problem of stresses in aelotropic circular discs of varying thickness rotating about a central axis (1949*a*, *b*). He has written a note on a case of forced torsional oscillation of a circular cylinder (1949*c*). H. M. Sengupta has contributed a number of papers on the bending of an elastic plate under certain distributions of load. (1948*a*, *b* ; 1949). B. R. Seth has written a monograph on two dimensional potential problems connected with rectilinear boundaries (1939). He has studied the bending plates with various types of boundaries (1945 ; 1947*d* ; 1948 ; 1949). He has also investigated transverse vibrations of rectilinear plates and stability of rectilinear plates (1947*a*, *b*). I. D. Seth has written a paper on the reflection and refraction of attenuated waves in semi-infinite elastic solid media (1941).

Bibliography

Ghosh, N. N. (1942). A matrix method of analysing strain and stress in hyperspace. *Bull. Calcutta math. Soc.*, **34**, 143-152.

Ghosh, S. (1942). On plain strain and plain stress in aelotropic bodies. *Ibid.*, **34**, 157-169.

——— (1943). Stress system in rotating aelotropic discs. *Ibid.*, **35**, 61-65.

——— (1944). On the divergence of the solution of a problem of plane strain. *Ibid.*, **36**, 51-58.

——— (1946). On generalized plane stress in an aelotropic plate. *Ibid.*, **38**, 61—66.

——— (1947). On a new function theoretic method of solving the torsion problem for some boundaries. *Ibid.*, **39**, 107-112.

——— (1948*a*). On the flexure of a beam whose cross-section is bounded partly by a straight line. *Ibid.*, **40**, 77-82.

——— (1948*b*). On the torsion and flexure of a beam whose cross-section is a quadrant of of a given area. *Ibid.*, **40**, 107-115.

Mitra, D. N. (1948). On the flexure problem for some boundaries. *Ibid.*, **40**, 173-182.

——— (1949*a*). Torsion and flexure of an isotropic elastic cylinder whose cross-section is a semi-cardioid. *Ibid.*, **41**, 28-40.

——— (1949*b*). On flexure functions of a semi-circular cylinder. *Ibid.*, **41**, 125-128.

——— (1949*c*). On the flexure problem of a limacon and some other boundaries. *Ibid.*, **41**, 153-158.

Sen, Bibhutibhusan (1942). Stresses in an infinite strip due to an isolated couple acting at a point inside it. *Ibid.*, **34**, 45-51.

——— (1943*a*). Stresses due to forces and couples acting in the interior of an infinite elastic slab placed on a rigid foundation. *Ibid.*, **35**, 13-20.

——— (1943*b*). Note on uniqueness of solution of problems of thin plates bent by normal pressures. *Ibid.*, **35**, 135-140.

Sen, Bibhutibhusan (1944*a*). Boundary value problems of circular discs under body forces, I. *Bull. Calcutta math, Soc.*, **36**, 58-62.
——— (1944*b*). Boundary value problems of circular discs under body forces, II. *Ibid.*, **36**, 83-86.
——— (1945*a*). Direct determination of stresses from the stress equations in some two-dimensional problems of elasticity, IV. Problems of wedges. *Phil. Mag.* (7), **36**, 66-72.
——— (1945*b*). Stresses in an infinite plate due to isolated forces and couples acting near a circular hole. *Ibid.*, (7), **36**, 211-216.
——— (1945*c*). Problems of thin plates with circular holes. *Bull. Calcutta math. Soc.*, **37**, 37-42.
——— (1946*a*). Two-dimensional boundary-value problems of elasticity. *Proc. roy. Soc.*, A, 1879, 87-101.
——— (1946*b*). Boundary value problems of heavy circular disc held in a vertical plane. *Phil. Mag.*, (7), **37**, 66-72.
——— (1946*c*). Note on the stresses in a semi-infinite plate produced by a rigid punch on the straight boundary. *Bull. Calcutta math. Soc.*, **38**, 117-120.
——— (1947). Direct determination of stresses in thin elastic plates having cavities of different shapes. *Ibid.*, **39**, 113-118.
——— (1948). Direct determination of stresses from the stress-equations in some two-dimensional problems of elasticity. Part V. Problems of curvilinear boundaries. *Phil. Mag.* (7), **39**, 992-1001.
——— (1949). Note on the deformation produced by some symmetrical distribution of variable loads on the plane boundary of a semi-infinite elastic solid. *Bull. Calcutta math. Soc.*, **41**, 77-82..
Sengupta, A. M. (1949*a*). Note on a simple case of forced torsional oscillation of a circular cylinder. *Bull. Calcutta math Soc.*, **41**, 47-48.
——— (1949*b*). Stresses in some aelotropic and isotropic circular discs of varying thickness rotating about the central axis. *Ibid.*, **41**, 129-139.
——— (1949*c*). Thermal stresses in isotropic circular discs of varying thickness rotating about a central axis. *Ibid.*, **41**, 199-207.
Sengupta, H. M. (1948*a*). On the bending of an elastic plate under certain distributions of load. *Ibid.*, **40**, 17-35.
——— (1948*b*). On the bending of an elastic plate under certain distributions of load. III. *Ibid.*, **41**, 53-63.
——— (1949). On the bending of an elastic plate. I. *Bull. Calcutta math. Soc.* **41**, 163-172.
Seth, B. R. (1944). Consistency equations of finite strain. *Proc. Indian Acad. Sci.*, A, **20**, 336-339.
——— (1945). Bending of an equilateral plate. *Proc. Indian Acad. Sci.*, A, **22**, 234-238.
——— (1946). Fine strain in aelotropic elastic bodies. II. *Bull. Calcutta math. Soc.* **38**, 39-44.
——— (1947*a*). Transverse vibrations of rectilinear plates. *Proc. Indian Acad. Sci.*, A, **25**, 25-28.
——— (1947*b*). Stability of rectilinear plates. *J. Indian math. Soc.* (N.S.), **100**, 13-16.
——— (1947*c*). Finite longitudinal vibrations. *Proc. Indian Acad. Sci.*, A, **25**, 151-152.
——— (1947*d*). Bending of clamped rectilinear plates. *Phil. Mag.*, (7), **38**, 292-297.
——— (1948). Bending of rectilinear plates. *Bull. Calcutta math. Soc.*, **40**, 36-40.
——— (1949). Bending of an elliptic plate with a confocal hole. *Quart. J. Mech.*, **2**, 177-181.
Seth, I. D. (1941). Reflection and refraction of attenuated waves in semi-infinite elastic solid media. *Proc. Indian Acad. Sci.*, A, **13**, 151-160.

FOURIER SERIES AND INTEGRALS

S. Minakshisundaram himself and also with another has contributed a series of papers to the theory of the Fourier expansion and theory of the differentiated terms of eigen-functions (1944*a*, *b*; 1945, 1946; 1947*b*; 1949). M. L. Misra has investigated the summability (A) and (C) of the conjugate series of a Fourier series and the successively derived series of the conjugate series of a Fourier series (1947*b*, *c*, *d*, *e*; 1946). He has also considered the jump of a function by its Fourier coefficients (1947*a*). F. C. Auluck has written a note on the Fermi-Dirac Functions (1942). R. P. Agarwal has discussed some new Kernals and functions which are self-reciprocal in the Hankel trnasform (1947). D. P. Banerji has studied the expansion of an arbitrary function in various special functions and series (1940, 1941*a*, *b*). K. Basu has written a note on Sonine's polynomials (1943).

K. Chandrasekharan himself and some in collaboration with Minakshisundaram has considered the summability of multiple Fourier series (1946, 1948*b*, *c*). In collaboration with Minakshisundaram he has obtained some results on double Fourier series (1947). They also have studied Fourier series in several variables (1948*a*). S. C. Dhar has discussed certain self-reciprocal functions (1940). H. C. Gupta has written a note on operational calculus and Hankel Transforms (1943*a*). He has obtained some kernels for the derivatives of self-reciprocal functions (1945*a*). He has given two theorems on self-reciprocal functions (1945*b*).

K. S. K. Iyenger has given new proofs of Mehler's formula and other theorems of Hermitian polynomials, of the formula or the generating function of Laguerre polynomials and other related formulae (1939*a*, *b*). He has studied the application of a certain Tauberian theorem to the convergence of Fourier series (1943*a*). He has also developed a new technique of convergence and summability tests for Fourier series (1943*b*). D. D. Kosambi has studied the zeros and closure of orthogonal functions (1942). P. C. Mital has studied the operational images of self-reciprocal functions (1942). S. C. Mitra has written a number of papers on reciprocal functions (1949*b*, *c*, *d*). Brij Mohan also has devoted a series of papers to the study of reciprocal functions (1940*a*, *b*, *c*; 1941*b*).

Ramnath Mahanty has investigated the summation of the integral conjugate to the Fourier integral of the finite type. In another paper he has determined the jump of a function by its Fourier series (1942). He has found a criterian for the absolute convergence of a Fourier series (1946). B. N. Prasad has given a review of the work on the summability of Fourier series and its conjugate series (1945*b*). Hari Shankar has made a study of certain self-reciprocal functions (1941*a*, *b*). A. Sharma has discussed a generalization of Legendre Polynomials (1948). In another paper he has made a study of certain theorems in operational calculus (1945).

A. N. Singh has written a note on divergent Fourier sine series (1943). U. N. Singh has investigated the strong summability of a Fourier series and its conjugate series (1947). S. D. Sinvhal has dealt with the Cesaro-non-summability of Fourier series (1946*b*, 1947). He has also studied the Riesz summability of Fourier series (1946*a*). He has also written a note on a divergent series of Legendre function (1943). R. S. Varma has studied a generalization of Laplace transform (1947). In another paper he has studied an inversion formula for the generalized Laplace transform (1949). He has discussed an infinite series involving the product of Bessel function and generalized Laguerre Polynomials (1940). He has also written a note on a certain self-reciprocal functions (1942).

BIBLIOGRAPHY

Agrawal, R. P. (1940). On the resultant of two functions. *Proc. Indian Acad. Sci.*, A, **27**, 141-146.

——— (1947). On some new kernels and functions self-reciprocal in the Hankel transform. *Proc. nat. Inst. Sci. India.* **13**, 305-318.

Auluck, F. C. (1942). The Fermi-Dirac functions. *Phil. Mag.*, (7), **33**, 159-160.

Banerji, D. P. (1940). On the application of integral equation to the expansion of an arbitrary function in a series of special functions. *Proc. nat. Acad. Sci. India*, A, **10**, 85-88.

——— (1941*a*). On some expansions containing Laguerre polynomials, and their expressions in terms of Whittaker's confluent hypergeometric functions. *Phil. Mag.*, (7), **32**, 84-85.

——— (1941*b*). On the properties of the functions which are self-reciprocal in Hankel's transform. *Bull. Calcutta math. Soc.*, **33**, 93-97.

——— (1949). On the convergence of certain lacunary trigonometric series. *Ibid.*, **41**, 86.

Basu, K. (1943). A note on Sonine's polynomials I. *Ibid.*, **35**, 21-32.

Bhatia, A. B. and Krishnan, K. S. (1948). Light scattering in homogeneous media regarded as reflexion from appropriate thermal elastic waves. *Proc. roy. Soc.*, A, **192**, 181-194

Bhatnagar, S. P. (1941). On the Fourier coefficients of a discontinuous function. *Proc. Edinb. math. Soc.*, (2), **6**, 231-256.

Bose, N. N. (1948). A theorem in operational calculus. *Phil. Mag.*, (7), **39**, 821-823.

Bose, S. K. (1949*a*). A study of the generalised Laplace integral II. *Bull. Calcutta math.* Soc., **41**, 59-67.

——— (1949*b*). A study of the generalised Laplace integral III. *Ibid.*, **41**, 68-76.

——— (1949*c*). On Laplace transorm of two variables. *Ibid.*, **41**, 173-178.

——— (1949*d*). Corrections to the paper on 'A study of the generalised Laplace integral'. *Ibid.*, **41**, 221-222.

Chandrasekharan, K. (1943). The absolute summability of series of eigenfunctions. *J. Indian math. Soc.*, (N.S.), **7**, 25-30.

——— (1944). On Stürm—Liouville series. *Ibid.*, **8**, 109-114

——— (1946). On multiple Fourier series. *Proc. Indian Acad. Sci.*, A, **24**, 229-232.

K. Chandrasekharan and Minakshisundaram, S. (1947). Some results on double Fourier series. *Duke math. J.*, **14**, 731-753.

——— (1948*a*). On Fourier series in several variables. *Ann. Math.*, (2), **49**, 991-1007.

——— (1948*b*). On the summation of multiple Fourier series I. *Proc. Indian math. Soc.*, (2), **50**, 212-222.

——— (1948*c*). On the summation of multiple Fourier series II. *Ibid.*, (2), **50**, 223-229.

Chandrasekharan, K. and Szesz, Otto (1948). On Bessel's summation. *Amer. J. Math.*, **70**, 709-729.

Chandrasekharan K. and Bochner ,S. (1949). Gauss summability of trigonometric integrals. *Amer. J. math.*, **71**, 50-59.

Chowla, S. and Auluck, F. C. (1940). On Weierstrass approximation theorem. Math. Stud., **8**, 78-79.

Dhar, S. C. (1940). On certain self reciprocal functions. *J. Indian math. Soc.*, (N.S.), **4**, 91-96.

Gupta, H. C. (1943*a*). Operational calculus and Hankel transforms. *Ibid.*, **7**, 117-128.

——— (1943*b*). Two self reciprocal functions. *Proc. nat. Acad. Sci. India*, A, **13**, 37-39.

——— (1945*a*). Some new kernels for the derivation of self-reciprocal functions. *Proc. Indian Acad. Sci.*, A,**22**, 228-233.

——— (1945*b*). A theorem on operational calculus. *J. India math. Soc.* (N.S.), **9**, 61-65.

——— (1945*c*). Two theorems on self-reciprocal functions. *Ibid.*, **9**, 66-68.

Iyengar, K. S. K. (1940). On a test for the completeness (L^2) of a normal orthogonal set and its applications. *J. Indian math. Soc.* (N.S.), **4**, 1-12.

——— (1943*a*). A Tauberian theorem and its application to convergence of Fourier series. *Proc. Indian Acad. Sci.*, A, **18**, 81-87.

——— (1943*b*). New convergence and summability tests for Fourier series. *Ibid.*, A, **18**, 113-120.

Kaushik, S. P. (1946). A theorem on the generalised Laplace's transform. *Proc. Benares math. Soc.* (N.S.), **8**, 7-10.

Kosambi, D. D. (1942). On the zeros and closure of orthogonal functions. *J. Indian math. Sec.*, **6**, 16-24.

Krishnan, K. S. (1948). A simple result in quadrature. *Nature*, **162**, 215.

——— (1948). On the equivalence of certain infinite series and the corresponding integral. *J. Indian math. Soc.* (N.S.), **12**, 79-88.

Minakshisundaram, S. (1944*a*). A note on the theory of Fourier series. *Proc. nat. Inst. Sci.* India, **10**, 205-215.

Minakshisundaram, S. (1944*b*). On the differentiated terms of eigenfunctions. *J. Indian math. Soc.* (N.S.), **8**, 75-78.
——— (1945). Note on Fourier expansion I. *J. Lond. math. Soc.*, **20**, 148-153.
——— (1946). Notes on Fourier expansions. II. *Ibid.*, **21**, 264-267.
——— (1949). Notes on Fourier expansions III. Fourier Steiltjes series. *Amer. J. math.*, **71**, 60-66.
Minakshisundaram S. and Szesz, Otto. (1947*a*). On absolute convergence of multiple Fourier series. *Trans. Amer. math. Soc.*, **61**, 36-53.
——— (1947*b*). A uniqueness theorem for eigenfunction expansions. *Proc. nat. Acad. Sci., Wash.* A., **33**, 76-77.
Misra, M. L. (1946). On the Cesāro summability of the successively derived series of the conjugate series of a Fourier series. *Bull. Calcutta math. Soc.*, **38**, 151-155.
——— (1947*a*). On the determination of the jump of a function by its Fourier coefficients. *Quart. J. math.*, **18**, 147-156.
——— (1947*b*). The summability (A) of the successively derived series of a Fourier series and its conjugate series. *Duke Math. J.*, **14**, 167-177.
——— (1947*c*). The summability (A) of the conjugate series of a Fourier series. *Ibid.*, **14**, 855-863.
——— (1947*d*). On the summability of the conjugate series of a Fourier series by logarthmic means. *Proc. nat. Inst. Sci. India* **13**, 157-168.
——— (1947*e*). On the non-summability (C, 1) of Fourier series. *Bull. Amer. math. Soc.*, 53, 510-514.
Mital, P. C. (1942). Operational images of self-reciprocal functions. *J. Indian, math. Soc.* (N.S.), **6**, 25-32.
——— (1943). On self-reciprocal functions. *Proc. nat. Acad. Sci. India* A, **13**, 42-43.
Mitra, S. C. (1947). On a method of summing divergent series. *J. Indian math. Soc.* (N.S.), **13**, 4-16.
——— (1949*a*). On the sum of a series analogous to Fourier series. *Ibid.*, **13**, 159-164.
——— (1949*b*). On certain self-reciprocal functions. *Bull. Calcutta math. Soc.*, **41**, 1-5.
——— (1949*c*). On pairs of functions which are reciprocal in Fourier-sine transforms. *Ibid.*, **41**, 49-52.
——— (1949*d*). Correction to paper 'On certain self reciprocal functions'. *Ibid.*, **41**, 222.
Mohan, Brij (1940*a*). Certain self-reciprocal functions. *Bull. Amer. math. Soc.*, **46**, 466-468.
——— (1940*b*). A pair of self-reciprocal functions. *Proc. Benares math. Soc.* (N.S.), 2, 39-42.
——— (1940*c*). A self-reciprocal function in the form of a series. *Bull. Calcutta math. Soc.*, **32**, 125-128.
——— (1941*a*). Self-reciprocal function involving Appell's functions. *J. Indian math. Soc.* (N.S.), **5**, 32-37.
——— (1941*b*). Formulae connecting self-reciprocal functions. *Indian J. Phys.* **15**, 337-341.
——— (1942). A class of Kernels. *J. Benares Hindu Univ. Silv. Jubilee Number*, 134-137.
Mohanty, Ramnath (1942). Determination of the jump of a function by its Fourier series. *Bull. Calcutta math. Soc.*, **34**, 99-103.
——— (1946). A criterion for the absolute convergence of a Fourier series. *Proc. Lond. math. Soc.*, (2), **51**, 186-196.
Prasad, B. N. (1945*a*). The summability of a Fourier series and its conjugate series. *Sci. & Cult.*, **10**, (9) supplement.
——— (1945*b*). The summability of a Fourier series and its conjugate series. *Proc. 32nd Sci. Congr.*, Nagpur, Part II, 24, pp.
Sahai, Basdeo (1940). On the summability of the conjugate series of the derived Fourier series. *Proc. nat. Acad. Sci. India*, A, **10**, 93-102.
Shanker, Hari (1941*a*). On functions which are Fourier series or cosine transforms of each other. *Proc. nat. Acad. Sci., India*, A, **11**, 73-77.
——— (1941*b*). A note on certain self-reciprocal functions. *J. Indian math. Soc.* (N.S.), **5**, 44-45.
———((1941*c*). On functions which are Hankel transforms of each other. *Ibid.*, **5**, 109-112.
Sharma, A. (1945). Some theorems in operational calculus. *Proc. Benares math. Soc.* (N.S.), **7**, 3-9.
——— (1948). On a generalisation of Legendre polynomials. *Bull. Calcutta math.*, **40**, 195-206.
Singh, A. N. (1943). On divergent Fourier's sine series. *Ibid.*, **5**, 41-44.
Singh, U. N. (1947). On the strong summability of a Fourier series and its conjugate series. *Proc. nat. Inst. Sci. India*, **13**, 319-325.
Sinvhal, S. D. (1943). On a divergent series of Legendre functions. *Proc. Benares math. Soc.* (N.S.), **5**, 37-39.
——— (1945). On the points of convergence of Singh's example. *Proc. Benares math. Soc.* (N.S.), **7**, 35-50.

Sinvhal, S. D. (1946*a*). On Riesz summability of Fourier series. *Ibid.*, **8**, (1), 11-24.
——— (1946*b*). On Cesàro non-summability of Fourier series. *Ibid.*, **8**, (2), 23-29.
——— (1947). On Cesàro non-summability of Fourier series II. *Ibid.*, **9**, 43-47.
Subba Rao, M. V. (1943). On generalized Legendre polynomials. *J. Indian math. Soc.* (N.S.), **7**, 96-101.
Tiwari, N. D. (1943). A theorem in operational calculus. *Proc. Benares math. Soc.* (N.S.), **5**, 33-36.
——— (1945). A theorem on the generalised Laplace's transform. *Ibid.*, **7**, 51-58.
Verma, R. S. (1940). An infinite series involving the product of Bessel's functions and generalised Laguerre polynomials. *Proc. Indian Acad. Sci.*, A, **12**, 532-534.
——— (1942). A self-reciprocal function. *Duke math. J.*, **9**, 259-261.
——— (1947). A generalisation of Laplace's transform. *Curr. Sci.*, **16**, 17-18.
——— (1949). An inversion formula for the generalised Laplace transform. *Proc. Edinb. math. Soc.*, (2), **8**, 126-127.

THEORY OF FUNCTIONS

S. M. Shah (1940; 1941*a, b, c*; 1942*a, b, c, d*; 1945*b*; 1946*a, b*; 1947*a, b*) has contributed a large number of papers to the theory of integral functions. P. D. Shukla (1940) has studied a non-differentiable function of Denjoy. D. D. Shukla (1943; 1945*a, b*; 1946) has discussed the question of the differentiability of certain functions. T. Vijayaraghavan (1947) has considered a power series that converges and diverges at everywhere dense sets of points on its circle of convergence. He has also studied certain functions represented by certain series (1942).

A. N. Singh (1941; 1943) has discussed certain functions without onesided derivatives. H. K. Sen (1940) has investigated certain properties of Darboux's theorem and its applications. C. T. Rajagopal (1941*a*) has given a proof of Hadamard's factorization theorem. He has also studied Caratheodory's inequality and allied results (1941*b*, 1947). He has written two papers on certain periodic functions (1945, 1948). In a note he has discussed some converse theorems of summability (1946). D. P. Banerji has discussed the generalizations of Weierstrass' non-differentiable functions (1946). In another paper he has studied the zeros of a non-differentiable function (1948).

K. Chandrasekharan (1941) has written a note on Hadamard's factorization theorem. V. Ganapathy Iyer (1941, 1942, 1946) has contributed a number of papers to the theory of integral functions. He has also considered the influence of zero on the magnitude of functions regular in an angle (1943). K. S. K. Iyengar (1940, 1941) has investigated a property of integral functions with real roots and of order less than two. Mahajani G. S. and Ram Behari (1947) have obtained an interesting result in the logarithmic expansion. In collaboration with V. R. Thiruvenkatachar, Mahajani has written a note on generalised mean value theorem (1950). S. Minakshisundaram (1940) has considered the roots of a continuous non-differentiable function. R. D. Misra (1940) has obtained a new non-differentiable function.

BIBLIOGRAPHY

Banerji, D. P. (1946). On a few generalisations of Weierstrass' non-differentiable functions. *Bull. Calcutta Math. Soc.*, **38**, 137-139.

——— (1948). On the zeros of a non-differentiable function. *Ibid.*, **40**, 145-146.

Bose, S. K. (1946). On the derivatives of integral functions. *J. Indian math. Soc.*, (N.S.), **10**, 77-80.

Chandrasekharan, K. (1941). On Hadamard's factorization theorem. *Ibid.*, **5**, 128-132.

——— (1943). On the canonical expression for a meromorphic function of finite order. *J. Madras Univ.*, B, **15.**

Ganapathy Iyer, V. (1942). A property of the maximum modulus of integral functions. *Ibid.*, **6**, 69-80.

——— (1946). On the translation numbers of integral functions. *Ibid.*, **10**, 17-28.

——— (1941). On periodic integral functions. *J. Indian math. Soc.* (N.S.), **5**, 1-17.

——— (1943). The influence of zeros on the magnitude of functions regular in an angle. *Ibid.*, **7**, 1-16.

Iyengar, K. S. K. (1940). A property of integral functions with real roots and of order less than two. *Proc. Indian Acad. Sci.*, A, **12**, 223-229.

——— (1941). A property of integral functions of order less than two with real roots. *Ann. Math.*, **2**, 42.

Keshava Menon, P. (1948). On a class of perfect sets. *Bull. Amer. math. Soc.*, **54**, 706-711.

Levi, F. W. (1950). On frequencies and semi-continuous functions. *Canad. J. Math.*, **2**, 32-43.

Mahajani, G. S. and Behari, Ram (1947). An interesting result in the logarithmic expansion. *J. Univ. Bombay* (N.S.), **15.**

Mahajani, G. S. and Thiruvenkatachar, V. R. (1950). A note on generalised mean value theorems. *Proc. Indian Acad. Sci.*, A, **31**, 124-129.

Minakshisundaram, S. (1940). On the roots of a continuous non-differentiable function. *J. Indian math. Soc.* (N.S.), **4**, 31-33.

Misra, R. D. (1940). On a new non-differentiable function. *Quart. J. Math. Oxford*, **11**.

Rajagopal, C. T. (1941*a*). A proof of Hadamard's factorization theorem. *Math. Student*, **9.**
——— (1941*b*). Caratheodory's inequality and allied results. *Ibid.*, **9.**
——— (1945). On periodic meromorphic functions. *J. Indian math. Soc.* (N.S.), **9.**
——— (1946). On converse theorems of summability. *Math. Gaz.*, **30**, 272-276.
——— (1947). Caratheodory's inequality and allied results II. *Math. Stud.*, **15**, 5-7.
——— (1948). A note on periodic integral function. *Duke math. J.*, **15.**
Rathnam, P. (1941). The derivatives of an algebraic function. *J. Indian math. Soc.* (N.S.), **5**, 145-147.
Sen, H. K. (1940). Darboux's property and its applications. *Proc. Benaras math Soc.* (N.S.), **2**, 17-23.
Sen Gupta, H. M. (1948). On continuous independent functions. *Quart. J. Math.*, **19**, 129-132.
Sastry, B. S. (1940). The limiting points of the set of the upper and the lower terms in Riemann integration. *Math. Stud.*, **8**, 148-150.
Singh, A. N. (1941). On function's without one-sided derivatives. *Proc. Benares math. Soc.* (N.S.), **3.**
——— (1943). On functions without one-sided derivatives II. *Ibid.*, **4.**
——— (1945). On infinite derivatives. *Fundam. Math.*, **33.**
14, 293-302.
Shah, S. M. (1940). A theorem on internal functions of integral order. *J. Lond. math. Soc.*, **15**, 23-31.
——— (1941*a*). A note on the classification of integral functions. *Math. Stud.*, **9**, 63-67.
——— (1941*b*). Note on a theorem of Polya. *J. Indian math. Soc.* (N.S.), **5**, 189-191.
——— (1941*c*). A theorem on integral functions of integral order II. *J. Indian math. Soc.* (N.S.), **5**, 179 ff.
——— (1942*a*). On integral functions of infinite order. *J. Univ. Bombay*, **11**, 9.
——— (1942*b*). On the relations between the lower order and the exponent of convergence of zeroes of an integral function. *Ibid.*, **11**, 10-13.
——— (1942*c*). Note on a theorem of Valiron and Collingwood. *Proc. nat. Acad. Sci. India*, **12**, 9-12.
——— (1942*d*). The lower order of the zeros of an integral function. *J. Indian math. Soc.* (N.S.), **6**, 63-68.
——— (1942*e*). The maximum term of an entire series. *Math. Stud.*, **10**, 810-812.
——— (1944). A note on the maximum modulus *J. Univ. Bombay* (N.S.), **13.**
——— (1945*a*). The lower order of the zeros of an integral function II. *Proc. Indian Acad. Sci.*, *A*, **21**, 162-164.
——— (1945*b*). On the singularities of a class of functions on the unit circle. *Bull. Amer. math. Soc.*, **52.**
——— (1946*a*). On the lower order of integral functions. *Ibid.*, **52.**
——— (1946*b*). On proximate orders of integral functions. *Ibid.*, **52**, 326-328.
——— (1947*a*). A note on the minimum modulus of a class of integral functions. *Ibid.*, **53**, 524.
——— (1947*b*). A note on the derivatives of integral functions. *Ibid.*, **53**, 1156-1163.
——— (1948*a*). A note on lower proximate orders. *J. Indian math. Soc.* (N.S.), **12**, 31-32.
——— (1948*b*). The maximum term of an entire series III. *Quart. J. Math.*, **19**, 220-223.
——— (1949*a*). A note on uniqueness sets for entire functions. *Proc. Indian Acad. Sci.*, A, **28**, 519-526.
——— (1949*b*). The maximum term of an entire series. *V*. J. *Indian math. Soc.* (N.S.), **13**, 60-64.
Shukla, D. D. (1943). On the differentiability of step functions. *Proc. nat. Acad. Sci. India*, A, **13**, 271-274.
——— (1945*a*). On the differentiability of functions. *Proc. Benares math. Soc.* (N.S.), **7.**
——— (1945*b*). On the differentiability of monotone functions. *Bull. Calcutta math. Soc.*, **37**, 9-14.
——— (1946). On the differentiability of an indefinite integral. *Ibid.*, **38**, 25-33.
Shukla, Parmeshwar Din (1940). On the derivatives of a function of Denjoy. *Proc. Benares math. Soc.*, (N.S.), **2**, 1-16.
Shukla, Uma Kant (1943). On a non-differentiable function. *Proc. Benares math. Soc.* (N.S.), **4.**
Subha Rao, M. V. (1942). Some elliptic function formulae, *Math. Stud.* **10**, 87-90.
Vijayaraghavan, T. (1942). On functions represented by certain series. *Quar. J. Math. Oxford*, **13**, 113-116.
——— (1947). A power series that converges and diverges at everywhere dense sets of points on its circle of convergence. *J. Indian math. Soc.* (N.S.), **11**, 69-72.

SPECIAL FUNCTIONS; INTEGRAL & FUNCTIONAL EQUATIONS

S. C. Dhar has written a note on the addition theorem of parabolic cylinder functions. He has given integral representations of Whittaker and Weber functions (1940, 1942). N. G. Shabde has contributed a series of papers on integrals involving Legendre and Bessel functions, confluent hypergeometric functions and Laguerre functions (1940*a*, *b*; 1941, 1943*a*, *b*; 1945, 1946). S. M. Shah has written a paper on real continuous solutions of algebraic difference equations (1947). Hari Shanker has written a paper on the expansion of the product of two parabolic cylinder functions of non-integral order (1940*a*). He has considered the integral representation of Weber's parabolic cylinder function and its expansion into an infinite series (1940*b*). He has devoted a number of papers to the study of integrals and expansions involving Weber's parabolic cylinder functions, Whittaker's confluent hypergeometric functions (1943*a*, *b*; 1944; 1948*a*, *b*; 1949). He has also written some papers on confluent hypergeometric functions and parabolic cylinder functions which are Hankel transforms of one another (1943*a*, 1946*a*, *c*).

N. A. Shastri has given results involving Bateman's polynomials, Angelesen's polynomial $\pi_n(x)$, Bessel functions of third order and confluent hypergeometric series and products of Laguerre polynomials (1940*a*, *b*, *c*; 1941*a*, *b*). S. Sinha studied infinite integrals involving Bessel functions and hypergeometric functions (1942*a*, *b*; 1944*a*, *b*). R. S. Varma has evaluated infinite integrals involving Whittaker's functions (1940*b*). He has also written a paper on integral representation for Whittaker function (1945). He has studied an infinite series of Weber's parabolic cylinder functions (1941*b*). B. R. Pasricha has evaluated some integrals involving Humbert functions and Whittaker functions (1943*a*, *b*, *c*).

B. Mohan has contributed a number of papers to the study of the properties of certain confluent hypergeometric functions (1941*b*, *c*, *e*, 1942*a*, 1943*b*). He has also evaluated infinite integrals involving Struve's functions (1942*b*, *c*, *e*). In another paper he has dealt with some infinite integrals involving Bessel functions (1942*f*). S. C. Mitra has studied certain infinite integrals involving Struve functions and parabolic functions (1946). He has discussed certain expansions involving Whittaker's M-Functions (1940). He has worked on certain transformations in generalised hypergeometric series (1943).

P. Keshava Menon has given a generalisation of Legendre functions (1941). H. C. Gupta has evaluated some infinite integrals involving Bessel functions (1943*b*). He has discovered the application of operational calculus to the evaluation of a certain class of definite integrals (1943*d*). D. P. Banerji has given the expansion of an arbitrary function in a series of toroidal functions of the second kind (1942*a*). He has also written a paper on the application of the operational calculus to the expansion of a function in a series of Legendre functions of the second kind (1942*b*). Haridas Bagchi has applied the method of difference equations to the summation of certain series involving Legendre and Bessel function (1940*b*). F. C. Auluck has considered the energy levels of an artificially bounded linear oscillator (1941).

Bibliography

Auluck, F. C. (1941). Energy levels of an artificially bounded linear oscillator. *Proc. nat. Inst. Sci. India*, **7**, 133-140.

Bagchi, Haridas (1940*a*). On a class of functional equations. *Sankhyā*, **5**, 71-78.

——— (1940*b*). The method of difference equations applied to the summation of certain series involving Legendre and Bessel functions. *J. Indian math. Soc.*, (N.S.) **4**, 13-24.

Banerji, D. P. (1940*a*). On some new integral relations between Bessels and Legendre functions of unrestricted degree. *J. Indian math. Soc*, (N.S.), **4**, 25-28.

Banerji, D. P. (1940*b*). On certain integrals and expansions containing Bessel and Legendre's associated functions. *Proc. nat. Acad. Sci. India*, A, **10**, 89-92.
——— (1941*a*). On infinite integrals containing parabolic cylinder functions. *Proc. Benares math. Soc.* (N.S.), **3**, 13-15.
——— (1941*b*). On the expansions and infinite integrals containing Whittaker's H-functions. *Proc. nat. Acad. Sci. India, A*, **11**, 84-86.
——— (1942*a*). The expansion of an arbitrary function in a series of toroidal functions of the second kind. *Amer. J. Math*, **64**, 72-80.
——— (1942*b*). On the application of the operational calculus to the expansion of a function in a series of Legendre's functions of the second kind. *Prog. of Edin. Math. Soc.*, **7**, 1-2.
——— (1943). On some infinite integrals. *Proc. Benares math. Soc.* (N.S.), **4**, 1-2.
Basu, K. (1943). A note on the oscillating rotator. *Indian J. Phys.*, **17**, 193-196.
——— (1945). A few applications of the well known transforms to functions involving Sonine's polynomials. *Proc. nat. Acad. Sci. India*, A, **14**, 181-190.
Bose, B. N. (1944*a*). Certain transformations in generalized hypergeometric series. *Bull. Calcutta math. Soc.*, **36**, 74-79.
——— (1944*b*). On certain integrals involving Legendre and Bessel functions. *Ibid.*, **36**, 125-132.
——— (1945). On some integrals involving Bessel functions. *Ibid.*, **37**, 77-80.
——— (1948*a*). On certain integrals involving Bessel function. *Ibid.*, **40**, 8-14.
——— (1948*b*). On some integrals involving E-functions. *Phil. Mag.*, (7), 39, 824-826.
Bose, S. K. (1945). On an integral equation associated with the equation for hydrogen atom. *Bull. Calcutta math. Soc.*, **37**, 51-61.
——— (1946*a*). On certain integrals involving Legendre and Bessel functions. *Ibid.*, **38**, 177-180.
——— (1946*b*). Integrals involving Legendre and Bessel functions. *Ibid.*, **38**, 181-184.
Chowla, S. and Auluck, F. C. (1940). Some properties of a function considered by Ramanujan. *J. Indian math. Soc.* (N.S.), **4**, 169-173.
Dhar, S. C. (1940). Note on the addition theorem of parabolic cylinder functions. *Ibid.*, **4**, 29-30.
——— (1942). Integral representations of Whittaker and Weber functions. *Ibid.*, **6**, 181-185.
Gupta, H. C. (1943*a*). Some infinite integrals involving Bessel functions. *Bull. Calcutta math. Soc.*, (N.S.), **35**, 7-11.
——— (1943*b*). Some infinite integrals. *Proc. Benares math. Soc.* (N.S.) **4**, 45-50.
——— (1943*c*). Some self-reciprocal functions. *Bull. Calcutta math. Soc.*, **35**, 67-70.
——— (1943*d*). Operational calculus and the evaluation of a certain class of definite integrals. *Proc. Benares math. Soc.* (N.S.), **5**, 1-16.
——— (1943*e*). Operational calculus and infinite integrals. *Proc. nat. Acad. Sci. India*, A, **13**, 225-231.
——— (1948). On operational calculus. *Proc. nat. Inst. Sci. India*, **14**, 131-156.
Kesava Menon, P. (1941). A generalization of Legendre functions. *J. Indian math Soc.*, (N.S.), **5**, 92-102.
——— (1943). Some inequalities involving the Γ and ζ functions. *Math. Stud.*, **11**, 10-12.
Mital, P. C. (1941). On integrals involving Legendre functions. *Proc. Benares math. Soc.* (N.S.), **3**, 17-21.
——— (1943). On self-reciprocal functions. *Ibid.*, **4**, 41-43.
Mitra, S. C. (1940). On certain expansions involving Whittaker's M-functions. *Proc. Edinb. math. Soc.*, (2), **6**, 157-159.
——— (1942*a*). On an extension of a theorem of Watson in generalised hypergeometric series. *J. Indian math. Soc.* (N.S.), **6**, 81-83.
——— (1942*b*). On a certain transformation about generalised hypergeometric series. *Ibid.*, **6**, 84.
——— (1943). On certain transformations in generalised hypergeometric series. *Ibid.*, **7**, 102-109.
——— (1946). On certain infinite integrals involving Struve functions and parabolic functions. *Proc. Edinb. math. Soc.*, (2) **7**, 171-173.
Mohan, B. (1941*a*). Some infinite integrals. *J. Indian math. Soc.*, (N.S.), **5**, 123-127.
——— (1941*b*). On certain confluent hypergeometric function. *Proc. nat. Acad. Sci. India*, **11**, 781-813.
——— (1941*c*). A confluent hypergeometric function *Ibid.*, **7**, 177-182.
——— (1941*d*). A class of infinite integral. I. *Sci. and Cult.*, **7**, 362-364.
——— (1941*e*). Properties of a certain confluent hypergeometric function. *Bull. Calcutta math. Soc.*, **33**, 99-103.
——— (1942*a*). Properties of a confluent hyper-geometric function. *Proc. nat.Inst. Sci. India*, **8**, 93-97.
——— (1942*b*). Infinite integral involving Struve's functions. II. *Proc. nat. Acad. Sci. India* **12**, 231-235.

Mohan, B. (1942*c*). Infinite integrals involving Struve's functions *Quart. J. Math.*, **13**, 40-47.
——— (1942*d*) A class of infinite integrals. II. *J. Indian math. Soc.* (N.S.), **6**, 98-101.
——— (1942*e*). Infinite integrals involving Struve's functions. III. *Bull. Calcutta math. Soc.*, **34**, 55-59.
——— (1942*f*). Infinite integrals involving Bessel functions II. *Ibid.*, **34**, 171-175.
——— (1943*a*). Some infinite integrals. *Proc. nat. Acad. Sci. India*, A, **13**, 171-178.
——— (1943*b*). On confluent hypergeometric functions. *Proc. Benares math. Soc.* (N.S.), **4**, 59-60.

Nath Sharma, Prithvi (1942). On the differential equation $f^r(x) = f(\frac{1}{x})$. *Math. Stud.*, **10**, 173-174.
Pasricha, B. R. (1943*a*). Some integrals involving Humbert function. *Proc. Indian Acad. Sci.*, A, **18**, 11-19.
——— (1943*b*). Some infinite integrals involving Whittaker functions. *J. Indian math. Soc.* (N.S.), **7**, 46-50.
——— (1943*c*). Some infinite integrals involving Whittaker's functions. *Proc. Benares math. Soc.* (N.S.), **4**, 61-69.
Shabde, N. G. (1940*a*). On some results involving confluent hypergeometric functions. *J. Indian math. Soc.* (N.S.), **4**, 151-157.
——— (1940*b*). On some results involving Legendre functions. *Bull. Calcutta math. Soc.*, **32**, 121-124.
——— (1941). On certain relation between Bessel and Laguerre functions. *Proc. Benares math. Soc.* (N.S.), **3**, 11.
——— (1943*a*). On some integrals involving Legendre functions. *Ibid.*, **4**, 3-8.
——— (1943b). On a definite-integral involving Legendre functions. *Ibid.*, **5**, 31-32.
——— (1945). Two integrals involving Legendre functions. *Ibid.*, **7**, (2), 51-53.
——— (1946). On some results involving Legendre functions. *Quart. appl. math.*, **4**, 302-305.
Shah, S. M. (1947). On real continuous solutions of algebraic difference equations. *Bull. Amer. math. Soc.*, **53**, 548-558.
Shah, S. M. and Sharma, V. C. (1948). Some properties of a function of Ramanujan. *J. Univ. Bombay* (N.S.), **17**, (3), A, 1-4.
Shanker, Hari (1940*a*). On the expansion of the product of two parabolic cylinder functions of non-integral order. *Proc. Benares math Soc.* (N.S.), **2**, 61-68.
——— (1940*b*). On integral representation of Weber's parabolic cylinder function and its expansion into an infinite series. *J. Indian math. Soc.* (N.S.), **4**, 34-38.
——— (1940*c*). On certain integrals and expansions involving Weber's parabolic cylinder functions. *Ibid.*, **4**, 158-166.
——— (1943*a*). On confluent hypergeometric functions which are Hankel-transforms of each other. *Ibid.*, **7**, 63-67.
——— (1943*b*). On some integrals and expansions involving Whittaker's confluent hypergeometric functions. *Proc. Benares math. Soc.* (N.S.), **4**, 51-57.
——— (1944). Some infinite integrals involving cylinder functions. *J. Indian math. Soc.* (N.S.), **8**, 57-60.
——— (1946*a*). On integral representations for the product of two Whittaker functions. *J. Lond. math. Soc.*, **22**, 112-115.
——— (1946*b*). On the Hankel transform of generalized hypergeometric functions. *Ibid.*, **21**, 194-198.
——— (1946*c*). Parabolic cylinder functions which are Hankel transforms of each other. *J. Ind. math. Soc.* (N.S.), **10**, 62-64.
——— (1948*a*). Some definite integrals involving confluent hypergeometric functions. *J. Lond. math. Soc.*, **23**, 44-49.
——— (1948*b*). Certain integral representations for Whittaker functions. *Proc. Camb. Phil. Soc.*, **44**, 453-455.
——— (1949). An integral equation for Whittaker's confluent hypergeometric functions. *Ibid.*, **45**, 482-483.
Sankara Pillai, K. (1945). An asymptotic expansion. *Math. Stud.*, **13**, 82-84.
Sharma, J. L. (1940). On the recurrence formulae of the generalized Lamé functions. *Ibid.*, **2**, 43-51.
——— (1943). Integral equations involving Lamé functions of complex degrees. *Ibid.*, **4**, 19-25.
Shastri, N. A. (1940*a*). Some results involving Bateman's polynomials. *Bull. Calcutta math. Soc.* **32**, 89-94.
——— (1940*b*). On Angelesen's polynomial $\pi_n(x)$. *Proc. Indian Acad. Sci.*, A, **11**, 312-317.
——— (1940*c*). Some results involving Angelesen's polynomial $\pi_n(x)$. *Ibid.*, A, **12**, 73-82.
——— (1941*a*). Some relations between Bessel's functions of third order and confluent hypergeometric functions. *Ibid.*, A, **13**, 521-525.
——— (1941*b*). Some integrals involving products of Laguerre polynomials. *Proc. Benares math. Soc.* (N.S.), **3**, 23-35.

Shastrí, N. A. (1944). Infinite series involving confluent hypergeometric functions. *Proc. Benares math. Soc.* (N.S.), **6**, 11-33.
Sinha, S. (1942*a*). A few infinite integrals. *J. Indian math. Soc.* (N.S.), **6**, 103-104.
——— (1942*b*). Some infinite integrals. *Bull. Calcutta math. Soc.*, **34**, 67-77.
——— (1944*a*). Infinite integrals involving Bessel functions of imaginary argument. *J. Indian math. Soc.* (N.S.), **8**, 21-26.
——— (1944*b*). A few integrals involving Bessel and hypergeometric functions. *Proc. Benares math. Soc.* (N.S.), **6**, 3-9.
Sircar, H. (1945). On the integrals $\int_0^x J_{K}(x)\,dx$ and $\int_0^x J_{-K}(x)\,dx$. *Bull. Calcutta math. Soc.*, **37**, 1-4.
Varma, R. S. (1940). Some infinite integrals involving Whittaker functions. *Proc. Benares math. Soc.* (N.S.), **2**, 81-84.
——— (1941*a*). On Humbert functions. *Ann. math.*, (2),**42**, 429-436.
——— (1941*b*). An infinite series of Weber's parabolic cylinder functions. *Proc. Benares math. Soc.* (N.S.), **3**, 37.
——— (1945). On integral representation for Whittaker functions. *J. Indian math. Soc.* (N.S.), **9**, 49.

GEOMETRY

A. Narsinga Rao (1940*a*) has written a large number of papers on Differential Geometry. He has generalised certain systems of circles, called Miquel-Clifford configurations. He has made studies in the Kasner's "turbine" geometry and the topology of the totality of line elements in the inversive plane. He has also written a paper on the metric geometry of a cyclic n-point (1945).

Ram Behari has studied ruled surfaces whose curved asymptotic lines can be determined by quadraluris (1942). He has also written a note on geodesic curvature (1947). Ram Behari (1940, 1941, 1944) has made important contributions to the theory of rectilinear congruences. He has studied the generalizations of the theorems of Malus-Dupin, Beltrann and Ribacour. He has discussed the five families of ruled surfaces through a line of a rectilinear congruence. He has also developed the theory of pitch of a congruence at a ray and has obtained expressions for it in some cases. He has investigated the properties of rectilinear congruences by using tensor analysis (1940). He has also written a note on normal rectilinear congruences (1944). Along with Ratan Shankar Misra, Ram Behari has found some formulae in rectilinear congruences (1949).

V. V. Narlikar and K. R. Karmakar (1949) have investigated the scalar invariants of a general gravitational metric. V. Rangachariar has also investigated some properties of rectangular hyperboloids (1941*a*). He has investigated into the nature and properties of conicoids of a pencil touching a given plane (1949*b*). Rangachariar and B. Singh (1945) have written a joint paper on the radical conic of two central conics.

D. D. Kosambi (1940) has worked in the field of Differential Geometry of higher dimensions and calculus of variations. He has written a note on the concept of isotropy in generalised path-spaces. He has dealt with Path equations admitting the Lorentz group (1941). In another paper he has studied the differential invariants of a two-index tensor (1949). R. N. Sen (1944 ; 1945 ; 1946) has written a number of papers on Parallelism in Riemannian Space. He has also contributed some papers to the discussion of parallel displacement and scalar product of vectors (1948 ; 1949*a*, *b*). N. Chatterji and P. N. Dasgupta (1940) have studied some congruence quadrics obtained from linear complexes of the irreducible system of two quaternary quadrics with two linear complexes.

Haridas Bagchi (1941*a*,*b* ; 1949) has investigated the properties of bicircular quartics, cylindrical and similar surfaces, and cyclides and hypercyclides. He has also written notes on the common tangents of a cubic and one of its sextactic conics and on the sextactic points of a cubic and its Hessian (1948*a*, 1949). B. Rammurti (1940) has given a geometrical proof of a theorem on spinors. Mohammad Shabbar (1941*a*) has discussed the existence of a metric for path-spaces admitting the Lorentz group. S. Chidambaram (1940) has written a note on a certain inscribed rectangle of a conic. B. R. Venkataraman (1941) has discussed some theorems in circle geometry (1941). C. N. Srinivasiengar (1940) has investigated some new properties of rectilinear congruences. A. C. Choudhury (1942*a*,*b* ; 1943 ; 1944*a*) has contributed a number of papers to the geometry of the web. P. Kesava Menon (1945) has given an extension of a theorem of Sterner.

Ram Mandan (1941*a*) has discussed the relation between a pencil and a range of quadrics. He has also studied the properties of mutually self polar tetrahedra (1941*b*). He has also written a note on symmetrical figure of circles and points (1945*a*). In another paper he has discussed the properties of Gauss-points in n-dimensional space (1949). N. N. Ghosh (1940*a*, 1945, 1948) has given a matrix treatment to the motion of a rigid body in hyperspace. He has also given a matrix theory of screws in hyperspace (1943).

V. R. Chariar (1945*a*) has considered the harmonic transversals of two pairs of qudrics and a straight line. He has also written a note on transversals which meet consecutive generators of a ruled surface of a constant angle (1945*c*). In another paper he has discussed the skewness of distribution of the generators of a ruled surface (1949). R. S. Misra (1949) has studied the geometrical properties of five families of ruled surfaces through a line of a rectilinear congruence.

Bibliography

Anand, Jagdish Gopal (1941). On a polar quadrics. *Bull. Calcutta math. Soc.*, **33**, 45-47.

Bagchi, Haridas (1941*a*). On bicircular quartics. *J. Indian math. Soc.*, (N.S.), **5**, 46-67.

——— (1941*b*). Cylindrical and kindred surfaces. *Bull. Calcutta math. Soc.*, **33**, 129-146.

——— (1948*a*). Note on the common tangents of a cubic and one of its sextactic conics. *Ibid.*, **40**, 135-139.

——— (1948*b*). Circles of double contact of a bi-circular quartic. *Ibid.*, **40**, 207-225.

——— (1949). Some novel properties of cyclides and hypercyclides. *J. R. Asiat. Soc. Sci.*, **15**, 121-130.

Behari, Ram (1940). A theorem on normal rectilinear congruences. *Proc. Indian Acad. Sci.*, A, **12**, 205-207.

——— (1941). Some properties of rectilinear congruences. *Proc. nat. Acad. Sci. India*, A, **11**, 64-66.

——— (1942). Ruled surfaces whose curved asymptotic lines can be determined by quadraluris. *Math. Stud.*, **10**, 59-63.

——— (1944). A note on normal rectilinear congruences. *Proc; nat. Acad. Sci. India*, A, **14**, 98-100.

——— (1946). The differential geometry of ruled surfaces. *Lucknow Univ. Stud.* No, **18**.

Chariar, V. R. (1944). On harmonic locus of two given quadrics. *Bull. Calcutta math. Soc.*, **36**, 41-44.

——— (1945*a*). On the harmonic transversals of two pairs of quadrics and a straight line. *J. Indian math. Soc.* (N.S.), **9**, 46-50.

——— (1945*b*). On a certain scroll associated with a net of quadrics. *Ibid.*, **9**, 105-108.

——— (1945*c*). Note on transversals which meet consecutive generators of a ruled surface at a constant angle. *Bull. Calcutta math. Soc.*, **37**, 133-136.

——— (1949). On the skewness of distribution of the generators of a ruled surface. *Proc. Indian Acad. Sci.*, A, **30**, 49-55.

Chariar, V. R. and Singh, B. (1949). On a certain rectilinear congruence. *J. Indian math. Soc.* (N.S.), **13**, 148-151.

——— (1947). A note on geodesic curvature. *Math. Stud.* **15**, 100-101.

——— (1949). Some formulate in rectilinear congruences. *Proc. nat. Inst. Sci. India*, **15**, 85-92.

Chatterji, N. and Dasgupta, P. N. (1940). On some congruence quadrics obtained from linear complexes of the irreducible system of two quaternary quadrics with two linear complexes. *Bull. Calcutta math. Soc.*, **32**, 43-50.

Chidambaram, S. (1940). On a particular inscribed rectangle of a conic. *Math. Stud.*, **8**, 143-147.

Choudhuri, A. C. (1942*a*). On the 4—web of the projective lines of curvature. *Bull. Calcutta math. Soc.*, **34**, 1-15.

——— (1942*b*). On the Steiner's polygons in a web. *J. Indian math. Soc.* (N.S.), **6**, 122-126.

——— (1942*c*). On a generalization of Thomsen's triangle in a web. *Bull. Calcutta math. Soc.*, **34**, 93-98.

——— (1942*d*). On a case of the cross ratio system of a 3-web. *Ibid.*, **34**, 177-181.

——— (1943). On a generalization of Reidemeister's figure in a web. *Ibid.*, **35**, 77-80.

——— (1944*a*). On 2-webs of curves in R_n. *J. Indian math. Soc.* (N.S.), **8**, 36-44.

——— (1944*b*). The invariants of webs of curves in R_n. *Ibid.*, **36**, 62-74.

Ghosh, N. N. (1940*a*). A matrix treatment of rigid body motion in hyperspace. *Bull. Calcutta math. Soc.*, **32**, 109-120.

——— (1940*b*). The tortuosity of a variety. *Ibid.*, **32**, 51-60.

——— (1941*a*). Dyadics and multidyadics in hyperspace. *Ibid.*, **33**, 105-117.

——— (1941*b*). The tortuosity of submanifolds of a variety. *Ibid.*, **33**, 187-195.

——— (1943). A matrix-theory of screws in hyperspace. *Ibid.*, **35**, 115-125.

——— (1945). Matrix treatment of a rigid body motion in complex space. *Ibid.*, **37**, 43-56.

——— (1948). Rigid rotation in hyperspace. *Ibid.*, **40**, 116-122.

Iyengar, K. V. (1942). A note on Poncelets problem. *J. Mysore Univ.*, B, **3**, 131-134.

Kerawala, S. M. (1941). Euclidean aspect of Hielmslev's geometry. *Proc. Benares math. Soc.*, **3**, 39-53.

Kerawala, S. M. (1947). Poncelet perism in two circles. *Bull. Calcutta, math. Soc.*, **39**, 85-105.
Kesava Menon, P. (1945). An extension of a theorem of sterner. *Math. Stud.*, **12**, 78-79.
Kosambi, D. D. (1940). The concept of isotropy in generalized path-spaces. *J. Indian math. Soc.*, (N.S.), **4**, 80-88.
——— (1941. Path equations admitting the Lorentz group II. *Ibid.*, **5**, 62-72.
——— (1945). Parallelism in the tensor analysis of partial differential equations. *Bull. Amer. math. Soc.*, **51**, 293-296.
——— (1946). Sur la differentiation covariante. *C. R. Acad. Sci., Paris*, **222**, 211-213.
——— (1947). Les invariants differentials d'un tenseur covariant a deux indices. *Ibid.*, **225**, 790-792.
——— (1949). The differential invariants of a two index tensor. *Bull. Amer. math. Soc.*, **55**, 90-94.
——— (1949). Lie rings in path space. *Proc. nat. Acad. Sci., Wash.*, **35**, 389-394.
Krishnaswami Ayyangar, A. A. (1941). The theorem of Feuerbach. *Math. Stud.*, **9**, 16-29.
Mandan, Ram (1941a). A relation between a pencil and a range of quadrics. *Proc. Indian Acad. Sci.*, A, **14**, 313-316.
——— (1941b). Properties of mutually self-polar tetrahedra. *Bull. Calcutta math. Soc.*, **33**, 147-155.
——— (1942). Umbilical projection. *Proc. Indian Acad. Sci.*, A, **15**, 16-17.
——— (1945a). A symmetrical figure of circles and points. *Proc. Lahore phil. Soc.*, **7**, 52.
——— (1945*b*). Distance in homogeneous coordinates. *Ibid.*, **7**, (1), 54-55.
——— (1946). Umbilical projection. *Proc. Indian Acad. Sci.*, A, **24**, 433-440.
——— (1949). Gauss-points in n-dimensional space. *Bull. Calcutta math, Soc.*, **41**, 6-8.
Misra, D. C. (1949). A property of closed regions. *Ibid.*, **41**, 83-85.
Misra, Ratan Shankar (1945). Some properties of rectilinear congruences obtained by tensor method. *Proc. Benares math. Soc.*, (N.S.), **7**, (2), 41-49.
——— (1946*a*). The five families of ruled surfaces through a line of rectilinear-congruence. *J. Indian math. Soc.*, (N.S.), **10**, 68-72.
——— (1946*b*). A note on Bianchi congruence. *Bull. Calcutta math. Soc.*, **38**, 141-142.
——— (1949). Five families of ruled surfaces through a line of rectilinear congruence. *J. Indian math. Soc.*, (N.S.), **13**, 81-84.
Mitra, Abha (1941). On affine regularity of polygons. *Ibid.*, **33**, 49-55.
Moghe, D. N. (1943). On isotropic spaces. *J. Univ. Bombay*, (N.S.), **12**, (3), 1-3.
Narayana Murthy, T. (1945). Feuerbach's theorem. *Math. Stud.*, **13**, 43-46.
——— (1947). On a system of circles associated with a symmedian of a triangle. *Math. Stud.*, **15**, 101-102.
Narlikar, V. V. and Bhedasgaokar, V. S. (1946). On the equations of certain transformations conserving the volume of a cuboid. *J. Banares Hindu Univ.*, **9**, 82-84.
Narlikar, V. V. and Karmarkar, K. R. (1949). The scalar invariants of a general gravitational metric. *Proc. Indian Acad. Sci.*, A, **29**, 91-97.
Narasinga Rao, A. (1940*a*). Studies in circle geometry. *Math. Stud.*, **8**, 53-72.
——— (1940*b*). Studies in turbine geometry. IV. The topology of oriented and non-oriented line elements in the inversive plane. *Proc. Indian Acad. Sci.*, A, **11**, 491-496.
——— (1945). On the metric geometry of a cyclic n-point I. *Math. Stud.*, **12**, 91-97.
——— (1947). Geometry as the study of space-structure. *Ibid.*, **15**, 89-92.
Narasingh Rao, A. and Venkataraman, M. (1945). On the Clifford and Grace chains. *Ibid.*, **12**, 98-101.
Padma, N. (1948). On the duality of linear tensors in affine space. *J. Indian math. Soc.* (N.S.), **12**, 107-116.
——— (1949). Skew tensors and region-complexes in Sn. *Ibid.*, **13**, 91-104.
Ramamurti, B. (1940). A geometrical proof of a theorem on spinors. *Proc. Indian Acad. Sci.*, A, **11**, 497-501.
——— (1942). On ten associated points in [4]. *Ibid.*, **16**, 191-192.
Ramesam, V. (1941). A further note on Feuerbach's theorem. *Math. Stud.*, **9**, 129-130.
Rangachariar, V. (1941*a*). On some properties of rectangular hyperboloids. *Ibid.*, **9**, 131-133.
——— (1941*b*). On the conicoids of a pencil touching a given plane. *Ibid.*, **9**, 158-160.
Rangachariar, V. and Singh, B. (1945). On the radical conic of two central conics. *Ibid.*, **12**, 86-87.
Rao, C. V. H. (1944). On a theorem on the plane cubic. *Bull. Calcutta math. Soc.*, **36**, 7-8.
Rao, R. (1949). Some coering theorems I. *Proc. Lond. math. Soc.*, (2), **51**, 232-264.
Satyanarayana, K. (1947). S-quadrics and isotomic polars. *Math. Stud.*, **15**, 13-15.
Seetharaman, V. (1945). The geometry of partial differential equations of the second order. *Proc. Indian Acad. Sci.*, A., **21**, 211-217.
Sen Gupta, B. K. (1945). On the aberrancy curve. *J. Indian math. Soc.* (N.S.), **9**, 77-79.
——— (1946). On the Cayleyan of a non-singular cubic. *Math. Stud.*, **14**, 20-22.
——— (1947). Propositions on polar reciprocals. *Bull. Calcutta math. Soc.*, **39**, 59-60.
——— (1949). Note on the closest contact of a conic. *Math. Stud.*, **17**, 38-40.
Sen, D. K. (1940). Pedal equations of a circle. *J. Univ. Bombay*, (N.S.), **9**, (3), 11-15.

Sen, R. N. (1944). On parallelism in Riemannian space. *Bull. Calcutta math., Soc.*, **36**, 102-107.

——— (1945). On parallelism in Riemannian space. II. *Ibid.*, **37**, 153-159.

——— (1946). On parallelism in Riemannian space. III. *Ibid.*, **38**, 161-167.

——— (1948). Parallel displacement and scalar product of vectors. *Proc. nat. Inst. Sci. India*, **14**, 45-52.

——— (1949*a*). Parallel displacement and scalar product of vectors. II. *Bull. Calcutta math. Soc.*, **41**, 41-46.

——— (1949*b*). Parallel displacement and scalar product of vectors. III. *Ibid.*, **41**, 113-120.

Shabbar, Mohammad (1941*a*). On the existence of a metric for path-spaces admitting the Lorentz group. *Proc. Indian Acad., Sci., A*, **13**, 203-210.

——— (1941*b*). Einstein spaces admitting the Loretz group. *Ibid.*, A, **14**, 133-138.

Srinivasiengar, C. N. (1940). Some properties of rectilinear congruences. *Ibid.*, A, **12**, 350-353.

——— (1941). The linear line-congruence. *J. Indian math. Soc.* (N.S.), **5**, 73-91.

——— (1942*a*). The resultant of wrenches on two given screws. *Bull. Calcutta math. Soc.*, **34**, (3), 131-138.

——— (1942*b*). On the quartic developable. *J. Indian math. Soc.* (N.S.), **6**, 127-130.

Suseela, M. (1946). Non-Desarguesian geometry. *Math. Stud.*, **14**, 1-13.

Venkataraman, B. R. (1941). On a new chain of theorems in circle-geometry. *J. Indian, math. Soc.* (N.S.)., **5**, 38-43.

——— (1945). Chain theorems in geometry. Chain of the Cox-Grace type. *Ibid.*, **9**, 1-28.

——— (1946). A new proof of the centre circle chain. *Ibid.*, **10**, 65-67.

THEORY OF GROUPS

F. W. Levi has contributed a series of papers to the Group Theory (1944*a, b, c, d*; 1945, 1946*a, b*). He has considered the problem of the number of generators of a free product (1941). He has given a discussion of the group theory in context to a problem of paper-folding and certain other problems (1942*c*). T. Venkatarayudu has studied normal co-ordinates of symmetric points group II and group S. He has also discussed the characters of the class of various forms in symmetric groups. He has written a paper on the Immanants of a matrix associated with a finite Abelian group (1943*a,b,c,d*, 1945*a,b*). Harish Chandra has developed the theory of Infinite irreducible representations of the Lorentz Group (1947). He has also considered the representations of Lie Algebra (1949; 1950). A. C. Chaudhuri has worked on Quasi-groups and non-associative systems I, II. (1948, 1949). R. C. Bose and S. Chowla have in a joint paper discussed a method of constructing a cyclic sub-group of order p+1 of the group of linear fractional transformation mod p (1945). K. G. Ramanathan discussed the problem of the product of the elements in a finite Abelian Group (1947). Ramadhar Misra has considered the Lattice sums of cubic crystals (1943).

Bibliography

Bose, R. C. and Chowla, S. (1945). On a method of constructing a cyclic sub-group of order p+1 of the group of linear fractional transformation mod. p. *Sci. & Cult.*, **10**, 558.

Choudhury, A. C. (1948). Quasi-groups and non-associative systems. I. *Bull. Calcutta math. Soc.*, **40**, 183-194.

——— (1949). Quasi-groups and non-associative systems. II. *Ibid.*, **41**, 211-219.

Harish Chandra (1947). Infinite irreducible representations of the Lorentz group. *Proc. roy. Soc., Canad.*, **139**, 372-401.

——— (1949). Faithful representations of Lie algebras. *Ann. Math.*, (2), **50**, 68-76.

——— (1950). On faithful representations of Lie groups. *Proc. Amer. math. Soc.*, **1**, 205-210.

Guha, U. (1946). On the endomorphic mappings {m} of a group. *Bull. Calcutta math. Soc.*, **38**, 101-107.

Kar, S. C. (1948). Zur Analyse der Lorentzgruppe in algebraischer Darstellung. *Ibid.*, **40**, 163-172.

Levi, F. W. (1940*a*). A problem on rigid motion. *Math. Stud.*, **8**, 1-10.

——— (1940*b*). The Commutator group of a free product. *J. Indian math. Soc.* (N.S.), **4**, 136-144.

——— (1940*c*). On a method of finite combinatories which applies to the theory of infinite groups. *Bull. Calcutta math. Soc.*, **32**, 65-68.

——— (1941). On the number of generators of a free product and a Lemma of Alexander Kurosch. *J. Indian math. Soc.* (N.S.), **5**, 149-155.

——— (1942*a*). Groups in which the commutator operation satisfies certain algebraic conditions. *Ibid.*, **6**, 87-97.

——— (1942*b*). Ordered groups. *Proc. Indian Acad. Sci.*, A, **16**, 256-263.

——— (1942*c*). On a theorem of group theory connected with a problem on paper folding and with some other problems solved and unsolved. *Math. Stud.*, **10**, 162-165.

——— (1943). Contributions to the theory of ordered groups. *Proc. Indian Acad. Sci.*, A, **17**, 199-201.

——— (1944*a*). Notes on group theory I & II. *J. Indian math. Soc.* (N.S.), **8**, 1-9.

——— (1944*b*). Notes on group theory III. Homogenous representation of groups. *Ibid.*, **8**, 44-56.

——— (1944*c*). On semigroups. *Bull. Calcutta math. Soc.*, **36**, 141-146.

——— (1944d). Notes on group theory IV-VI. *J. Indian math. Soc.* (N.S.), **8**, 78-91.

——— (1945). Notes on group theory VII. The idempotent residue classes and the mappings {m}. *Ibid.*, **9**, 37-42.

——— (1946*a*). On semigroups II. *Bull. Calcutta math. Soc.*, **38**, 123-124.

——— (1946*b*). The ring of endomorphisms for which every sub-group of an Abelian group is invariant. *J. Indian math. Soc.* (N.S.), **10**, 29-31.

Misra, Rama Dhar (1943). Lattice sums of cubic crystals. *Proc. nat. Acad. Sci. India*, A, **13**, 275-283.

Ramanathan, K. G. (1947). On the product on the elements in a finite Abelian group. *J. Indian math. Soc.* (N.S.), **11**, 44-48.
Venkatarayudu, T. (1943*a*). Normal co-ordinates of symmetric point groups. *Proc. Indian Acad. Sci.*, A, **17**, 50-54.
——— (1943*b*). Normal co-ordinates of symmetric point groups II. *Proc. Indian Acad. Sci.*, *A*, **17**, 75-78.
——— (1943*c*). The character table of a subgroup of the symmetric group of degree S. *Ibid.*, A, **17**, 79-82.
——— (1943*d*). The characters of the classes (n-k, k) of the symmetric group of degree n. *J. Indian math. Soc.* (N.S.), **7**, 42-45.
——— (1945*a*). Characters of the classes of the form (n_1, n_2, n_3) in symmetric groups. *Proc. Indian Acad. Sci.*, A, **22**, 42-45.
——— (1945*b*). Immanants of a matrix associated with a finite Abelian group, *Ibid.*, A, **21**, 103-104.

HYDROMECHANICS

A. C. Banerji and R. S. Varma have written a paper on the motion of a compressible ellipsoid in a viscous fluid (1942). M. Ray studied turbulent flow in a paper (1947). He has also discussed the development of liquid motion due to an impulse (1949). S. K. Roy has considered the motion of a local vortex round a disturbed corner (1941). In another paper he has studied the equilibrium of a local vortex (1943). He has also written a paper on a case of slow viscous flow (1942). B. R. Seth has obtained some solutions in viscous flow by the method of superposition of effects (1942). He has discussed the stress-strain velocity relations in equations of viscous flow (1944). In collaboration with Qabul Chand Gupta he has applied the method of images to waves in canals (1940).

H. Sircar has commented on H. Poncin's problem (1942). V. R. Thruvenkatachar has investigated the motion of a thin airfoil in supersonic stream. He has also studied compressible shear flow. He has evolved an analogue of Blasius' formula in subsonic compressible flow (1948, 1949). S. L. Malurkar has made a study of the dynamics of thunderstorms (1943). S. Chandrasekhar has developed the theory of statistical and isotropic turbulence (1949*a, b, c*). Ram Ballabh has written a number of papers on superposable motions in incompressible fluid (1941, 1942, 1943*a, b*; 1945). Shanti Ram Mukherji has contributed a number of papers to the solution of viscous motion in incompressible fluid under various laws of viscosity variance (1942*a, b, c,* 1943*a, b*).

Bibliography

Ballabh, Ram (1941). Superposable motions in a heterogeneous incompressible fluid. *Proc. Benares math. Soc.* (N.S.), **3**, 1-9.

——— (1942). On superposability. *J. Indian math. Soc.* (N.S.), **6**, 33-40.

——— (1943*a*). Steady uniplaner superposable fluid motions. *J. Indian math. Soc.* (N.S.), **7**, 36-41.

——— (1943*b*). On two-dimensional self-superposable fluid motions. *Proc. Benares math. Soc.* (N.S.), **4**, 27-31.

——— (1943*c*). Fluid motion of the type $\xi_1 = \lambda_2 n_2$ etc. and $\xi_2 = \lambda_1 n_1$ etc. *Proc. nat. Acad. Sci.*, A, **13**, 151-158.

——— (1945). On fluid motions superposable on an irrotational motion. *Proc. Benares math. Soc.* (N.S.), **7**, 11-15.

Banerji, A. C. and Varma, R. S. (1942). On the motion of a compressible ellipsoid in a viscous fluid. *Ibid.*, **4**, 77-94.

Chandrasekhar, S. (1949*a*). The theory of statistical and isotropic turbulence. *Phys. Rev.*, (2), **75**, 896-897.

——— (1949*b*). On the decay of isotropic turbulence. *Phys. Rev.*, (2), **75**, 1454-1455.

——— (1949*c*). On the decay of isotropic turbulence. *Phys. Rev.*, (2), **76**, 158.

Malurkar, S. L. (1943). Dynamics of thunderstorms. *Proc. Indian Acad. Sci.*, A, **18**, 20-27.

Mukherjee, Santi Ram (1942*a*). Motion of an incompressible fluid with varying coefficient of viscosity given by $\mu = \mu_0 = \epsilon_1 x$ for positive values of x. I. *Proc. nat. Acad. Sci. India*, A, **12**, 121-130.

——— (1942*b*). Motion of an incompressible fluid with varying coefficient of viscosity, given by $\mu = \mu_0 + \epsilon_1 x$ for positive values of x. II. *Ibid.*, A, **12**, 151-160.

——— (1942*c*). Motion of an incompressible fluid with varying viscosity coefficient given by $\mu = \mu_0 + \epsilon_1 x$ for positive values of x. III. *Ibid.*, A, **12**, 161-167.

——— (1943*a*). Motion in incompressible fluid of variable design. *Ibid.*, A, **13**, 1-18.

——— (1943*b*). Motion in incompressible fluid of variable density. II. *Ibid.*, A, **13**, 135-150.

Nath Sharma, Pirthvi (1943). Elliptic sources and vortices. *Proc. Benares math. Soc.*, (N.S.), **4**, 33-38.

Ray, M. (1947). On a type of non-stationary turbulent wave. *Bull. Calcutta math. Soc.*, **39**, 139-142.

——— (1949). Development of liquid motion due to an impulse. *Ibid.*, **41**, 179-182.

Roy, S. K. (1941). Motion of a local vortex round a disturbed corner. *Proc. Benares math. Soc.* (N.S.), **3**, 71-93.

——— (1942). On a case of slow viscous flow. *Bull. Calcutta math. Soc.* **34**, 109-127.

——— (1943). Equilibrium of a local vortex. *Proc. Benares math. Soc.* (N.S.), **5**, 45-67.

Seth, B. R. (1942). Viscous solutions obtained by superposition of effects. *Proc. Indian Acad. Sci.*, A, **16**, 193-195.

——— (1944). On the stress-strain velocity relations in equations of viscous flow. *Proc. Indian Acad. Sci.*, A, **20**, 329-335.

Seth, B. R. and Gupta, Qabul Chand (1940). Method of images applied to waves in canals. *Proc. Benares math. Soc.* (N.S.), **2**, 25-31.

Sircar, H. (1942). On H. Poncin's problem. *Bull. Calcutta math. Soc.*, **34**, 210-213.

Thruvenkatachar, V. R. (1948). The analogue of Blasiu's formulae in subsonic compressible flow. *Proc. nat. Inst. Sci. India*, **14**, 339-342.

——— (1949*a*). Compressible shear flow. I. Thin airfoil in steady supersonic flow. *Proc. nat. Inst. Sci. India*, **15**, 219-231.

——— (1949*b*). Compressible shear flow. II. Non-steady motion of thin airfoil in supersonic stream. *Proc. nat. Inst. Sci. India*, **15**, 233-240.

THEORY OF NUMBERS

S. Chowla (1944*b, c*) has written a large number of papers on various problems in the Number theory. He has given solutions of a certain problem of Erdös and Turan in additive number theory. He has found a new proof of a theorem of Siegel. He has also written a paper in the K-analogue of a result in the theory of the Riemann Zeta function (1943*a*). He has discussed the Waring's problem (mod *p*) (1943*b*). He has discovered a new property of biquadratic residues (1944*f*). Hansraj Gupta (1942*b, c*; 1946*a, b*) has developed the partition theory in a number of papers and prepared tables of partitions. He has studied symmetric functions in the theory of integral numbers. He has investigated some properties of generalized combinatory functions (1941*b*). He has written a note on some idiosyncratic numbers of Ramanujan (1941*c*). He has studied the congruence properties of σ(n) and τ (n). (1945, 1946*c*, 1948*a*). He has also prepared a table of value of τ (n) (1947*a*). He has written several papers on various other aspects of the number theory.

D. R. Kaprekar (1946) has given some interesting applications of diagonalization method. P. Keshava Menon (1940) has obtained several congruence theorems (1940). He has also given some theorems on residues (1943*a*). He has written two papers on arithmetic functions (1942*c*, 1943*b*). He has studied some congruence properties of ϕ function (1946*a*). He has also given a generalization of Wilson's theorem (1945*a*). In another paper he has obtained some generalizations of the divisor function (1945*b*).

Krishnaswami Ayyangar (1940, 1941) has studied the theory of the nearest square continued fraction. In collaboration with P. R. Kaprekar he has written and discussed the congruence properties of Ramanujan's function τ (n) (1946). He has written two papers on some non-Ramanujan congruence properties of the partition function (1948; 1949). A. M. Mian and S. Chowla have studied the differential equations satisfied by certain functions (1944*a, b*).

S. S. Pillai (1940*a, b, e, f, g, h, i*; 1941*a, b*) has worked on normal numbers, consecutive integers and prime numbers. He has written a number of papers on Waring's problems with powers of primes (1940*c*, 1944*c*). He has studied the problem of lattice points in a right angled triangle (1943*a, b*). He has written a note on the smallest primitive root of a prime (1944*a*). K. G. Ramanathan (1943*b*, 1944*a*, 1945*a, b*, 1947) has written a series of papers on the congruence properties of Ramanujan's function τ (n). He has also studied certain of Ramanujan's Trigonometric sums C_m(n) and their applications (1943*a*, 1944*b*). K. Sambasiva Rao has written two papers on the representation of a number as the sum of the *k*th power of a prime and *l*th power of free integer (1940*a*). C. S. Venkataraman (1946*a, b, c, d, e*; 1949*a*) has studied multiplication functions.

T. Vijayraghavan (1940*a*) has written a note on decimals of irrational numbers. In collaboration with S. Chowla he has studied the complete factorization (mod *p*) of the cyclotomic polynomials of order p^2-1. (1944). They have also given short proofs of the theorems of Bose and Sinjer (1945). He has written a paper on the fractional parts of powers of a number (1948).

R. P. Bambah has contributed a number of papers (some with others) to the study of congruence properties of Ramanujan's function τ (n) (1946, 1946*a, b*; 1947). D. P. Banerji (1942*a*) has worked on congruence properties of Ramanujan's function τ (n). He has obtained some new properties of the arithmetic function τ (n) (1942*b*). He has discussed the solution of a Waring's problem (1942*c*). He has written two papers on some formulae in analytical theory of numbers (1944*a, b*).

Bibliography

Auluck, F. C. (1940). On Waring's problem for biquadrates. *Proc. Indian Acad. Sci.*, A, **11**, 437-450.

——— (1942). An asymptotic formula for $p_x(n)$. *J. Indian math. Soc.* (N.S.), **6**, 113-134.

Auluck, F. C., Chowla, S. and Gupta, H. (1942). On the maximum value of the number of partitions of n into k-parts. *Ibid.*, **6**, 105-112.

Bambah, R. P. (1946*a*). On integer cuberoots of the unit matrix. *Math. Stud.*, **14**, 69-70.

——— (1946*b*). Two congruence properties of Ramanujan's function $\tau(n)$. *J. Lond. math. Soc.*, **21**, 91-93.

——— (1946*c*). On a function of Ramanujan. *Ibid.*, **12**, 433.

——— (1947). Ramanujan's function $\tau(n)$—a congruence property. *Ibid.*, **53**, 764-765.

Bambah, R. P. and Chowla, S. (1946*a*). Some new congruence properties of Ramanujan's functions τ (n). *Math. Stud.*, **14**, 24-26.

——— (1946*b*). A congruence property of Ramanujan's function τ (n). *Proc. nat. Inst. Sci. India*, **12**, 431-432.

——— (1946*c*). On complete primative residue sets. *Bull. Calcutta math. Soc.*, **38**, 113-116.

——— (1946d). On integer roots of the unit matrix. *Sci. & Cult.*, **12**, 105.

——— (1947*a*). A new congruence property of Ramanujan's function $\tau(n)$. *Bull. Amer. math. Soc.*, **53**, 768-769.

——— (1947*b*). Congruence properties of Ramanujan's function τ (n). *Ibid.*, **53**, 950-955.

——— (1947*c*). A note on Ramanujan's function $\tau(n)$. *Quart. J. Math.* **18**, 122-123.

——— (1947*d*). On numbers which can be expressed as a sum of two squares. *Proc. nat. Inst. Sci. India*, **13**, 101-103.

——— (1947*e*). On integer roots of the unit matrix. *Ibid.*, **13**, 241-246.

——— (1947*f*). The residue of Ramanujan's function τ(n) to the modulus 2^8. *J. Lond. math. Soc.*, **22**, 140-147.

Bambah, R. P., Chowla, S. and Gupta, H. (1947). A congruence property of Ramanujan's function $\tau(n)$. *Bull. Amer. math. Soc.*, **53**, 766-767.

Bambah, R. P., Chowla, S., Gupta, H. and Lahiri, D. B. (1947). Congruence properties of Ramanujan's function $\tau(n)$ *Quart. J. Math* **18**, 143-146.

Banerji, D. P. (1942*a*). Congruence properties of Ramanujan's function τ (n). *J. Lond. math. Soc.*, **17**, 144-145.

——— (1942*b*). On the new congruence properties of the arithmetic function $\tau(n)$. *Proc. nat. Acad. Sci. India*, A, **12**, 149-150.

——— (1942*c*). On the solution of the 'easier' Waring's problem. *Bull. Calcutta math. Soc.*, **34**, 197-199.

——— (1943). On the rational solution of the Diophantine equation $ax^n - by^n = k$. *Proc. Benares math. Soc.* (N.S.), **5**, 29-30.

——— (1944*a*). On some formulae in analytical theory of numbers. *Bull. Calcutta math. Soc.*, **36**, 49-50.

——— (1944*b*). On some formulae in analytical theory of numbers II. *Ibid.*, **36**, 107-108.

——— (1945*a*). On the application of the congruence property of Ramanujan's function to certain quaternary form. *Ibid.*, **37**, 24-26.

——— (1945*b*). On a theorem in the theory of partition. *Ibid.*, **37**, 113-114.

——— (1947). On the divisor of numbers. *Ibid.*, **39**, 57-58.

Bose, R. C. (1942*a*). On some new series of balanced incomplete block design. *Bull. Calcutta math. Soc.*, **34**, 17-31.

——— (1942*b*). An affine analogue of Sinjer's theorem. *J. Indian math. Soc.* (N.S.), **6**, 1-15.

Bose, R. C. and Chowla, S. (1945). On the construction of affine difference set. *Bull. Calcutta math. Soc.*, **37**, 107-112.

Bose, R. C. and Chowla, S. and Rao, C. R. (1944). On the integral order (mod. p) of quadratics x^2+ax+p, with applications to the construction of minimum functions for GF (p^2) and to some number theory results. *Bull. Calcutta math. Soc.*, **30**, 153-174.

——— (1945*a*). A chain of congruences. *Proc. Lahore phil. Soc.*, **7**, (1), 1-53.

——— (1945*b*). Minimum functions in Galois fields. *Proc. nat. Acad. Sci. India*, A, **15**, 191-192.

Bose, R. C and Nair, K. R. (1941). On complete sets of latin squares. *Sankhyā*, **5**, 361-382.

Chakraberti, M. C. (1940). On the limit points of a function connected with the three-square problem. *Bull. Calcutta math. Soc.*, **32**, 1-6.

Chowla, S. (1943*a*). On the K-analogue of a result in the theory of the Riemann zeta function. *Proc. Benares math. Soc.*, **5**, 23-27.

——— (1943b). On the Waring's problem (mod. p). *Proc. nat. Acad. Sci. India*, A, **13**, 195-220.

——— (1944*a*). There exists an infinity of 3-combination of primes in A. P. *Proc. Lahore phil. Soc.*, **6**, (2), 15-16.

Chowla, S. (1944*b*). Solutions of a problem of Erdös and Turan in additive number theory *Proc. nat. Acad. Sci. India*, A, **14**, 1-2.

——— (1944*c*). A property of biquadratic residues. *Ibid.*, A, **14**, 45-46.

——— (1944*d*). Contribution to the theory of the construction of balanced incomplete block designs used in the statistical tables of Fisher and Yates. *Proc. Lahore phil. Soc.*, **6**, (2), 10-12.

——— (1944*e*). The cubic character of 2 (mod p). *Ibid.*, **6**, (2), 12.

——— (1944*f*). *Proc. Lahore phil. Soc.*, **6**, (2), 17-23.

——— (1944*g*). On $g(k)$ in Waring's problem. *Proc. Lahore phil. Soc.*, **6**, 16-17.

——— (1945*a*). A formula similar to Jacobsthel's for the explicit value of x in $p=x^2+y^2$ where p is a prime of the form $4k+1$. *Proc. Lahore phil. Soc.*, **7**, 2.

——— (1945*b*). A cubic character of 2 (mod p). *Ibid.*, **7**, 1*p*.

——— (1945*c*). Outline of a new method for proving results of elliptic function theory. *Ibid.*, **7**, 3*p*.

——— (1945*d*). Contributions to the theory of the construction of balanced incomplete block designs. *Math. Stud.*, **12**, 82-85.

——— (1945*e*). On difference set. *J. Indian math. Soc.* (N.S.), **9**, 28-31.

——— (1945*f*). On quintic equations soluble by radicals. *Math. Stud.*, **13**, 84.

——— (1946). A note on multiplication functions. *Proc. nat. Insti. Sci. India*, **12**, 429-430.

——— (1947a). On an unsuspected real zero of Epstein's zeta functions. *Proc. nat. Inst. Sci. India*, **13**, (4), 1, P.

——— (1947*b*). A theorem in analytic number theory. *Ibid.*, **13**, 97-99.

——— (1947*c*). Modular equations as solutions of algebraic differential equations of the sixth order. *Ibid.*, **13**, 169-170.

——— (1947*d*). On the class number of the corpus P ($\sqrt{-k}$). *Proc. nat. Inst. Sci. India*, **13**, 197-200.

——— (1947*e*). On a problem of analytic number theory. *Ibid.*, **13**, 231-232.

——— (1947*f*). Note on a certain arithmatical sum. *Ibid.*, **13**, 233.

———(1947*g*). On a theorem of Walfisz. *J. Lond. math. Soc.*, **22**, 136-140.

——— (1947*h*). Proof of a theorem of Lerch and P. Keshava Menon. *Math. Stud.*, **15**, 4.

——— (1949*a*). Improvement of a theorem of Linnik and Walfisz. *Proc. Lond. math. Soc.*, (2), **50**, 423-429.

——— (1949*b*). An improvement of a theorem of Linnik and Walfisz. *Proc. nat. Inst. Sci. India*, **15**, 81-84.

——— (1949*c*). The last entry in Gaus's diary. *Proc. nat. Acad. Sci., Wash.*, **35**, 244-246.

——— (1949*d*). On difference sets. *Proc. nat. Acad. Sci., Wash.*, **35**, 92-94.

——— (1950). A new proof of a theorem of Siegel. *Ann. Math.*, (2), **51**, 120-122.

Chowla, S. and Singh, Daljit (1945*a*). A perfect difference set of order 18. *Proc. Lahore phil. Soc.*, **7**, 52.

——— (1945b). A perfect difference set of order 18. *Math. Stud.* **12**, 85.

Chowla, S., and Nazir, A. R. (1946). Numbers representable by a ternary quadratic form II. *Math. Stud.*, **14**, 23.

Chowla, S. D. and Vijayaraghavan, T. (1947). On the largest prime divisors of numbers. *J. Indian math. Soc.* (N.S.), **11**, 31-37.

Chowla, S. D. and Todd, John (1949). A density of reducible integers. *Canad. J. math.*, **1**, 297-299.

Chowla, S and Selbej, A. (1949). On Epstein's zeta function I. *Proc. nat. Acad. Sci., Wash.*, A, **35**, 371-374.

Gangadharan, K. S. (1945). On some identities and a generalization of Wilson's theorem. *Math. Stud.*, **12**, 75-77.

Gupta, Hansraj (1940*a*). Symmetric Functions in the Theory of Integral Numbers. Lucknow Univ. Studies, No. 14, Allahabad Law Journal Press, Allahabad, India, vii & 105 pp.

——— (1940*b*). On the absolute weight of an integer. *Proc. Indian Acad. Sci.*, A, **12**, 60-62.

——— (1940*c*). Waring's problem for powers of primes II. *J. Indian math. Soc.* (N.S), **4**, 71-79.

——— (1941*a*). Some properties of generalised combinatory functions. *Ibid.*, **5**, 27-31.

——— (1941*b*). On numbers of the from 4^a $(8b+7)$. *Ibid.*, **5**, 192-202.

——— (1941*c*). An important congruence. *Proc. Indian Acad. Sci. A*, **13**, 85-86.

——— (1941*d*). Some idiosyncratic numbers of Ramanujan. *Ibid.*, A, **13**, 519-520.

——— (1942*a*). On an asymptotic formula in partitions. *Ibid.*, A, **16**, 101-102.

——— (1942*b*). A formula in partitions. *J. Indian math. Soc.* (N.S.), **6**, 115-117.

——— (1942*c*). An inequality in partitions. *J. Univ. Bombay*, **11**, 16-18.

——— (1942*d*). On the class numbers of binary quadratic forms. *Rev. Univ. Tucumán*, A, **3**, 283-299.

——— (1943*a*). On residue chains. *Math. Stud.*, **11**, 54-55.

——— (1943*b*). Congruence properties of τ (n). *Proc. Benares math. Soc.*, **5**, 17-22.

——— (1943*c*). A formule for L (n). *J. Indian math. Soc.* (N.S.), **7**, 68-71.

Gupta, Hansraj (1943*d*). On the maximum values of $p_K(n)$ and $\pi_K(n)$. *Ibid.*, **7**, 72-75.

——— (1945). Congruence properties of $\sigma(n)$. *Math. Stud.*, **13**, 25-29.

——— (1946*a*). A note on the parity of p(n). *J. Indian math. Soc.* (N.S.), **10**, 32-33.

——— (1946*b*). An asymptotic formula in partitions. *Ibid.*, **10**, 73-76

——— (1946*c*). A congruence property of $\tau(n)$. *Proc. Indian Acad. Sci.*, A, **24**, 441-442.

——— (1947*a*). A table of values of $N_2(t)$. *Proc. nat. Inst. Sci. India*, **13**, 35-63.

——— (1947*b*). A table of values of $\tau(n)$. *Ibid.*, **13**, 201-206.

——— (1947*c*). On *nth* power residues. *Quart. J. Math.*, **18**, 253-256.

——— (1948*a*). A congruence relation between $\tau(n)$ and $\delta(n)$. *J. Indian math. Soc.* (N.S.), **9**, 59-60.

——— (1948*b*). The vanishing of Ramanujan's function $\tau(n)$. *Curr. Sci.*, **17**, 180.

——— (1948*c*). On $N_q(r)$ in the Tarry-Escott problem. *Proc. nat. Inst. Sci. India*, **14**, 335-336.

——— (1949*a*). On a conjecture of Miller. *J. Indian math. Soc.* (N.S.), **13**, 85-90.

——— (1949*b*). A solution of the Tarry-Escott problem of degree *r*. *Proc. nat. Inst. Sci. India*, **15**, 37-39.

——— (1950). On a problem of Erdös. *Amer. math. Mon.*, **57**, 326-329.

Hukum Chand (1945). Proof of some well known theorems in continued fractions. *Math. Stud.*, **13**, 98-101.

Kadian, Shersingh (1946). Numbers representable by a ternery quadratic form I. *Ibid.*, **14**, 22-23.

Kaprekar, D. R. (1945). A peculiar gap filling process for powers of $(9)^n$. *J. Univ. Bombay* (N.S.), **13**, (5), A, 1.

——— (1946). Some interesting application of diagonalisation method. *Ibid.*, **15**, (3), A, 7-9.

——— (1948). Demlo numbers. Khareswada Devlali, India, 114 pp.

Kesava Menon, P. (1940). A theorem on congruence. *Math. Stud.*, **8**, 156-158.

——— (1942*a*). Some congruence theorems. *Proc. Indian Acad. Sci.*, A, **16**, 95-100.

——— (1942*b*). Multiplicative functions which are functions of the g.c.d. and l.c.m. of the arguments. *J. Indian math. Soc.* (N.S.), **6**, 137-142.

——— (1942*c*). Transformations of arithmetic functions. *Ibid.*, **6**, 143-152.

——— (1943*a*). Some theorems on residues. *Proc. Indian Acad. Sci.*, A, **17**, 107-113.

——— (1943*b*). On arithmetic functions. *Proc. Indian Acad. Sci.*, A, **18**, 88-99.

——— (1943*c*). Identities in multiplicative functions. *J. Indian math. Soc.* (N.S.), **7**, 58-62.

——— (1945*a*). Some generalization of the divisor function. *J. Indian math. Soc.* (N.S.), **9**, 32-36.

——— (1945*b*). A generalization of Wilson's theorem. *Ibid.*, **9**, 79-88.

——— (1945*c*). Transformation of products of I-functions. *Ibid.*, **9**, 93-105.

——— (1945*d*). On the equation $x_1^3+x_2^3=Y_1^3+Y_2^3$. *Math Stud.*, **13**, 52-54.

——— (1946*a*). Some congruence properties of the ϕ-function. *Proc. Indian Acad. Sci.*, A, **24**, 443-447.

——— (1946*b*). On the equation $ax^2 \pm by^2 = cz^2$. *Math. Stud.*, **14**, 77-80.

——— (1946*c*). Some Diophantine equations. *Ibid.*, **14**, 65-68.

Krishnaswami Ayyangar, A. A. (1940). Theory of the nearest square continued fraction. *J. Mysore Univ.*, A, **1**, 21-32.

——— (1941). Theory of the nearest square continued fraction. *Ibid.*, A, **1**, 97-117.

Krishnaswami Ayyangar, A. A. and Kaprekar, P. R. (1945). Demelofication of Fibonacei numbers. *J. Univ. Bombay* (N.S.), **14**, part 5, A, 6-10.

Lahiri, Debabrata (1946). Congruence properties of Ramanujan's function $\tau(n)$. Sci. *& Cult.*, **12**, 52.

Lahiri, D. B. (1946*a*). On the type of series involving the partition function with applications to certain congruence relations. *Ibid.*, **38**, 125-132.

Lahiri, D. B. (1946*b*). On Ramanujan's function $\tau(n)$ and the divisor function $\sigma_K(n)$. I. *Bul. Calcutta math. Soc.*, **38**, 193-206.

——— (1946*a*). On the type of series involving the partition function with applications to certain congruence relations. *Ibid.*, **38**, 125-132.

——— (1947). On Ramanujan's functions $\tau(n)$ and the divisor function $\sigma K(n)$. II. *Ibid.*, **39**, 33-52.

——— (1948). Some non-Ramanujan congruence properties of the partition function. *Proc. nat. Inst. Sci. India*, **14**, 337-338.

——— (1949). Further non-Ramanujan congruence properties of the partition function. *Sci. & Cult.*, **14**, 336-337.

Majumdar, Kulendra Narayan (1948). On the divisors of $a\alpha^n + b\beta^m$. *Bull. Calcutta math. Soc.*, **40**, 15-16.

Mian, A. M. and Chowla, S. (1944*a*). The differential equations satisfied by certain functions. *J. Indian math. Soc.* (N.S.), **8**, 27-28.

Mian, A. M. and Chowla, S. (1944*b*). The differential equations satisfied by certain functions. *Proc. Lahore phil. Soc.*, **6**, 9-10.

——— (1944*c*). On the B_2—sequences of Sidon. *Proc. nat. Acad. Sci. India*, **A**, **14**, 3-4.

Misra, Rama Dhar (1947). On lattice sums for closest packing crystals. *Proc. Benares math. Soc.*, (N.S.), **4**, 109-112.

Narasimhamurti, V. (1940). On Demlo numbers. *Math. Stud.*, **8**, 34-37.

——— (1941). On Waring's problem for 8th, 9th and 10th powers. *J. Indian math. Soc* (N.S.), **5**, 122.

Nasir, Abdur Rahman (1946). On a certain arithmetic function. *Bull. Calcutta math. Soc.*, **38**, 140.

Prasad, Serveshwar, and Chariar, V. R. (1946) On certain Diophantine equations. *Patna Univ. J*, **2**, 66-71.

Nasir, A. R. (1947). Residue of $\sigma_K(n)$ modulii 2. *Amer. math. Mon.*, **54**, 274-275.

——— (1947). Properties of three numbers in a G.P. *Bull. Calcutta math. Soc.*, **39**, 79-81.

Pillai, S. S. (1940*a*). On *m* consecutive integers I. *Ibid.*, A, **11**, 6-12.

——— (1940*b*). Generalisation of a theorem of Mangoldt. *Ibid.*, A, **11**, 13-20.

——— (1940c). On m consecutive integers II. *Ibid.*, A, **11**, 73-80.

——— (1940*d*). On Waring's problem $g(6)=73$. *Ibid.*, A, **12**, 30-40.

——— (1940*e*). Waring's problems with indices $\geqslant$n. *Ibid.*, A, **12**, 41-45.

——— (1940*f*). A note on Gupta's previous paper. *Ibid.*, A, **12**, 63-64.

——— (1940*g*). On normal numbers. *Ibid.*, A, **12**, 179-184.

——— (1940*h*). On a linear Diophantine equation. *Ibid.*, A, **12**, 199-201.

——— (1940*i*). On Waring's problem with powers of primes. *Ibid.*, A, **12**, 202-204.

——— (1941*a*). On the sum function connected with primitive roots. *Ibid.*, A, **13**, 526-529.

——— (1941*b*). On m consecutive integers III. *Ibid.*, A, **13**, 530-533.

———(1941*c*). On algebraic irrationals. *Ibid.*, A, **15**, 173-176.

——— (1942*a*). On a problem in Diophantine approximation. *Ibid.*, **A**, **15**, 177-189.

——— (1942*b*). On the congruence property of the divisor function. *J. Indian math. Soc.* (N.S.), **6**, 118-119.

——— (1942*c*). On the divisors or a^n+1. *Ibid.*, **6**, 120-121.

——— (1943a). Lattice points in a right angled triangle II. *Proc. Ind. Acad. Sci.*, **A**, **17**, 58-61.

——— (1943*b*). Lattice points in a right angled triangle III. *Ibid.*, A, **17**, 62-65.

——— (1943*c*). On $\sigma_{-1}(n)$ and $\phi(n)$. *Ibid.*, A, **17**, 67-70.

——— (1943*d*). Highly abundant numbers. *Bull. Calcutta math. Soc.*, **35**, 141-156.

——— (1944*a*). On $n^X - L^Y = b^y \pm a^x$. *J. Indian math. Soc.* (N.S.), **8**, 10-13.

——— (1944*b*). On the smallest primitive root of a prime. *Ibid.*, **8**, 14-17.

——— (1944*c*). On Waring's problem with powers of primes. *Ibid.*, **8**, 18-20.

——— (1944*d*). Highly composite numbers of the ϵth order. *Ibid.*, **8**, 61-74.

——— (1944*e*). Bertrand's postulate. *Bull. Calcutta math. Soc.*, **36**, 97-99.

——— (1944*f*). On *m* consecutive integers IV. *Ibid.*, **36**, 99-101.

——— (1945*a*). On the equation $2^x-3^y=2^X+3^Y$ *Ibid.*, **37**, 15-20.

——— (1945*b*). Bertrand's postulates and correction to the above. *Ibid.*, **37**, 27.

——— (1945*c*). Universal rational functions. *Proc. nat. Acad. Sci.*, *Wash.*, **31**, 317-319.

Prasad, A. V. (1948). Note on a theorem of Hurwitz. *J. Lond. math. Soc.*, **23**, 169-171.

Rai, T. (1947). On a problem of additive theory of numbers. *Math. Stud.*, **15**, 25-28.

Radhakrishna Rao, C. (1944). Extension of the difference theorems of Sinjer and Bose. *Sci. & Cult.*, **10**, 57.

——— (1945). Finite geometries and certain derived results in theory of numbers. *Proc. nat. Inst. Sci. India*, **11**, 136-149.

——— (1946). Difference sets and combinational arrangements derivable from finite geometries. *Ibid.*, **12**, 123-135.

Ramanathan, K. G. (1943*a*). On Ramanujan's trigonometrical function. $C_m(n)$. *J. Madras Univ.*, B, **15**, 1-9.

——— (1943*b*). Congruence property of $\sigma(n)$, the sum of the divisors of *n*. *Math. Stud.*, **11**, 33-35.

——— (1943*c*). Multiplicative arithmetic functions. *J. Indian math. Soc.* (N.S.), **7**, 111-116.

——— (1944*a*). Congruence properties of Ramanujan's function $\tau(n)$. *Proc. Ind. Acad. Sci.*, A, **19**, 146-148.

——— (1944*b*). Some applications of Ramanujan's trigonometrical sum $C_m(n)$. *Proc. Ind. Acad. Sci.*, **A**, **20**, 62-69.

——— (1945*a*). Congruence properties of Ramanujan's function $\tau(n)$ II. *Ibid.*, (N.S.), **9**, 55-59.

Ramanathan, K. G. (1945*b*). Congruences properties of δ (n). *Math. Stud.*, **13**, 30.

——— (1947). Congruence properties of δ_a(N). *Proc. Ind. Acad. Sci.*, B, **25**, 314-321.

——— (1950). Identities and congruences of the Ramanujan's type. *Canad. J. Math.*, **2**, 168-178.

Ramaswami, V. (1948*a*). On the number of positive integers $< X$ and free of prime divisors $> X^c$, *Sci. & Cult.*, **13**, 465.

——— (1948*b*). On the number of integers $\leqslant X$ and free of prime divisors $> X^c$ *Ibid.*, **13**, 503.

——— (1948*c*). On the number of integers $\leqslant X$ and free of prime divisors $> X^c$, and problem of S. S. Pillai. *Ibid.*, **13**, 503.

——— (1948*d*). Sequences satisfying $a_1 = 1$, and $(r+1)^{-1}a_r \leqslant a_r - a_{r+1}$ *Math. Stud.*, **16**, 31-33.

——— (1949*a*). The number of positive integers $\leqslant x$ and free of prime divisors $> x^c$, and a problem of S. S. Pillai. *Duke math. J.*, **16**, 99-109.

——— (1949*b*). On the number of positive integers less than x and free of prime divisors greater than x^c. *Bull. Amer. math. Soc.*, **58**, 1122-1127.

Ramaswami, V. and Sambasiva Rao, K. (1945). On the probability that two *k*th power-free integers belonging to an assigned arithmetic progression should be prime to one another. *J. Ind. math. Soc.* (N.S.), **9**, 88-92.

Sambasiva Rao, K. (1940*a*). On the representation of a number as the sum of the kth power of a prime and on 1th power-free integer. *Proc. Ind. Acad. Sci.*, A, **11**, 429-436.

——— (1940*b*). Generalisation of a theorem of Pillai Selberg. *Ibid.*, A, **11**, 502-504.

——— (1940*c*). A correction. *J. Ind. math. Soc.* (N.S.), **4**, 125.

——— (1941). On Waring's problem for smaller powers. *Ibid.*, **5**, 117-121.

Sambasiva Rao, K. and Nagabhushanam, K. (1943). A note on Vinogradow numbers. *Ibid.*, A, **17**, 71-74.

Sastry, S. (1940). On Tarry's problem. *J. Indian math. Soc.* (N.S.), **4**, 167-168.

——— (1947). On equal sums of like powers. *Math. Stud.*, **15**, 29-32.

Sastri, S. and Rai, T. (1948). On equal sums of like powers. *Math. Stud.*, **16**, 18-19.

Sathe, L. G. (1943*a*). On a congruence property of the divisor function I. *J. Ind. math. Soc.* (N.S.), **7**, 143-145.

——— (1943*b*). On a congruence property of the divisor function II. *Ibid.*, **7**, 146-151.

——— (1945). On a congruence property of the divisor function. *Amer. J. Math.*, **67**, 397-406.

Sen Gupta, H. M. (1947). On Ramanujan function τ(n). *Math. Stud.*, **15**, 9-10.

Srinivasan, A. K. (1945). Residual types of partitions of 'O' into four cubes. *Math. Stud.*, **13**, 47-48.

——— (1948). Practical numbers. *Curr. Sci.*, **17**, 179-180.

Srinivasan, M. S. (1949). Theory of Pharinxes. *J. Univ. Bombay* (N.S.), **18**, (3), A, 4-22.

Suryanarayana Rao, B. (1945). On numbers which are the sum or difference of two cubes. *Math. Stud.*, **13**, 51-53.

Venkatachalam Iyer, R. (1947). Prime numbers whose reciprocals have the maximum recurring period. *J. Univ. Bombay* (N.S.), **15**, (5), A, 9-10.

Venkata Rama Ayyar, M. (1940). A note on three allied problems of Ramanujan. *J. Mysore Univ.*, B, **1**, 109-114.

Venkatarama Ayyar, and Bhimsena Rao, M. (1940). Types of solutions of $x^3+y^3+z^3=1$ in integers (I). *J. Ind. math. Soc.* (N.S.), **4**, 47-70.

Venkataraman, C. S. (1946*a*). A new identical equation for multiplicative functions of two arguments and its applications to Ramanujan's sum C_M(N). *Proc. Ind. Acad. Sci.*, A, **24**, 519-529.

——— (1946*b*). On some remarkable types of multiplicative function. *J. Ind. math. Soc.* (N.S.), **10**, 1-12.

——— (1946*c*). Further applications of the identical equation to Ramanujan's C^M(N) and Kronecker's function ρ(M, N). *Ibid.*, **10**, 57-61.

——— (1946*d*). The ordinal correspondence and certain classes of multiplicative functions of two arguments. *Ibid.*, **10**, 81-101.

——— (1946*e*). A theorem on residue and its bearing on multiplicative functions with a modulus. *Math. Stud.*, **14**, 59-62.

——— (1949*a*). Classification of multiplicative functions of two arguments based on the identical equation. *J. Ind. math. Soc.* (N.S.), **13**, 17-22.

——— (1949*b*). A generalisation of Euler's ϕ function. *Math. Stud.*, **17**, 34-36.

Vijayaraghavan, T. (1940*a*). On decimals of irrational numbers. *Proc. Ind. Acad. Sci.*, A, **12**, 20.

Vijayaraghavan, T. (1940*b*). The general rational solution of some Diophantine equation of the form $\sum_{r=1}^{k+1} A_r X_r^{n_r} = 0$. *Ibid.*, A, **12**, 284-289.

——— (1940*c*). On the fractional parts of the powers of a number I. *J. Lond. math. Soc.*, **15**, 159-160.

——— (1942). On the fractional parts of the powers of a number III. *Ibid.*, **17**, 137-138.

Vijayaraghavan, T. (1948). On the fractional parts of powers of a number IV. *J. Indian math. Soc.* (N.S.)., **12**, 33-39.

Vijayaraghavan, T. and Chowla, S. (1944). The complete factorization (mod. p) of the cyclotomic polynomials of order p^2-1. *Proc. nat. Acad. Sci. Ind.*, A, **14**, 101-105.

——— (1945). Short proofs of theorems of Bose and Sinjer. *Ibid.*, A, **15**, 194.

——— (1948). On complete residue sets. *Quart. J. Math., Oxford*, **19**, 193-199.

RELATIVITY MECHANICS

V. V. Narlikar has studied the two body problem in Einstein's new relativity theory (1941*a*). He has investigated whether Einstein's theory of new relativity is consistent with the geodesic postulate (1941*b*). He has discussed the gravitational equations of motion in relativity (1941*c*). In collaboration with P. C. Vaidya he has considered non-static electro-magnetic fields with spherical symmetry (1947, 1948). With K. R. Karmarkar as the co-author he has investigated the conditions of plane orbits in classical and relativistic fields (1946*a*). They have also studied the geodesic form of Schwarzschild's external solutions (1946*b*). Narlikar and Ayodhya Prasad have written a paper on the Doppler effect in the field of a thick spherical shell (1949). They have also studied the canonical co-ordinates in general relativity (1948). Narlikar and Ramji Tewari have written two papers on Einstein's generalised theory of gravitation (1949*a*, *b*, *c*; 1949*c*). In collaboration with K. P. Singh, V. V. Narlikar has discussed gravitational fields of spherical symmetry and Weyl's conformal curvature tensor (1950).

K. R. Karmarkar has investigated an important particular case of the problem of equivalence (1947*b*). He has obtained a new theorem in the transformability of line element into the spherically symmetric form (1947*c*). D. N. Moghe has discussed the theory of a system of receding particles having a tendency to approach the central mass (1940). He has also examined the kinematical theory and general relativity (1942*a*). Sir Shah Sulaiman has suggested a modification in a postulate of a Relativity theory (1940*a*). S. K. Roy has pointed out certain inconsistencies in the Mathematical theory of a new relativity developed by Sir Shah Sulaiman (1940). Shah Sulaiman has given a reply to Roy's objection and maintains that there are no inconsistencies (1940*b*). P. C. Vaidya has studied the external field of a radiating star in general relativity (1943). He has also written a paper on spherically symmetric line elements used in general relativity (1945).

BIBLIOGRAPHY

Datta Majumdar, S. (1946*a*). On the relativistic analogue of Earnshaw's theorem on the stability of a particle in a gravitational field. *Bull. Calcutta math. Soc.*, **38**, 85-92.

——— (1946*b*). A note on a class of solutions of Einstein's electro-static field equation. *Sci. & Cult.*, **12**, 295.

——— (1947*a*). Note on a class of solutions of Einstein's field equations in an electro-static field. *Ibid.*, **12**, 344.

——— (1947*b*). A class of exact solutions of Einstein's field equations. *Phys. Rev.* (2), **72**, 390-398.

Ghosh, J. (1948). A type of solutions of Einstein's gravitational equations. *Bull. Calcutta math. Soc.*, **40**, 45-47.

Jaiswal, J. P. (1945*a*). On the electric potential of a single electron in gravitational fields. I. *Proc. Benares math. Soc.* (N.S.), **7**, 17-25.

——— (1945*b*). On the electric potential of a single electron in gravitational fields. II. *Ibid.*, **7**, No. 2, 1-8.

Kar, S. C. (1948). Die Lorentz transformations und ihr physikalischer inhelt. *Bull. Calcutta math. Soc.*, **40**, 83-106.

Karmarkar, K. R. (1947*a*). On stationary line-elements. *Proc. nat. Inst. Sci. India*, **13**, 151-157.

——— (1947*b*). An important particular case of the problem of equivalence. *Bull. Calcutta math. Soc.*, **39**, 30-32.

——— (1947*c*). A new theorem in the transformability of a line-element into the spherically symmetric form. *Proc. Indian Acad. Sci.*, A, **26**, 52-55.

——— (1948). Gravitational metrices of sphereical symmetry and class one. *Proc. Indian Acad. Sci.*, A, **27**, 56-60.

Moghe, D. N. (1940). On the theory of a system of receding particles having a tendency to approach the central mass. *Proc. nat. Inst. Sci. India*, **6**, 225-235.

Moghe, D. N. (1942*a*). Kinematical theory and general relativity. *J. Univ. Bombay* (N.S.), **11**, (3), 1-5.

——— (1942*b*). A note on isotropic manifolds. *Ibid.*, **11**, (3), 6-8.

Narlikar, V. V. (1941*a*). The two-body problem in Einstein's new relativity. *Proc. nat. Inst. Sci. India*, **7**, 237-246.

——— (1941*b*). The consistency of Einstein's new relativity with the geodesic postulate. *Curr. Sci.*, **10**, 164-165.

——— (1941*c*). The gravitational equations of motion in relativity. *Proc. Indian Acad. Sci.*, A, **14**, 187-195.

Narlikar, V. V. and Vaidya, P. C. (1942). The equations of fit in general relativity. *Curr. Sci.*, **11**, 390-391.

——— (1947). A spherically symmetrical non-static electromagnetic field. *Nature*, **159**, 642.

——— (1948). Non-static electro-magnetic fields with spherical symmetry, *Proc. Nat. Inst. Sci. India*, **14**, 53-54.

Narlikar, V. V., Patwardhan, G. K. and Vaidya, P. C. (1943). Some new relativistic distribution of radian symmetry. *Proc. nat. Inst. Sci. India*, **9**, 229-236.

Narlikar, V. V. and Karmarkar, K. R. (1946*a*). Conditions of plane orbits in classical and relativistic fields. *Proc. Indian Acad. Sci.* A, **24**, 451-455.

——— (1946*b*). Geodesic form of Schwarzschild's external solutions. *Nature*, **157**, 515-516.

——— (1946*c*). On the curious solution of relativistic field equation. *Curr. Sci.* **15**, 69.

Narlikar, V. V. and Singh, K. P. (1948). On a gravitational invariant. *Proc. nat. Inst. Sci. India*, **14**, 121-123.

Narlikar, V. V. and Prasad, Ayodhya (1948). Canonical coordinates in general relativity. *Bull. Calcutta, math. Soc.*, **40**, 123-128.

——— (1949). The Doppler effect in the field of a thick spherical shell. *Proc. Indian Acad. Sci.*, A, **30**, 181-183.

Narlikar, V. V. and Tewari, Ramji (1949*a*). On Einstein's generalised theory of gravitation. *Proc. Nat. Inst. Sci. India* **15**, 73-79.

——— (1949*b*). On Einstein's generalized theory of gravitation. II. *Ibid.*, **15**, 249-261.

——— (1949*c*). A particular case in Einstein's generalized theory of gravitation. *Phys. Rev.*, (2), **76**, 868-869.

Narlikar, V. V. and Sing, K. P. (1950). Gravitational fields of spherical symmetry and Weyl's conformal curvature tensor. *Phil. Mag.*, (7), **41**, 152-156.

Patwardhan, G. K. and Vaidya, P. C. (1943). Relativistic distributions of matter of radial symmetry. *J. Univ. Bombay* (N.S.), 12, (**3**), 23-36.

Racine, C. (1941*a*). Contribution to the relativistic problem of *n* bodies. I. *J. Indian math. Soc* (*N.S.*), **5**, 156-164.

——— (1941*b*). Contribution to the relativistic problem of *n* bodies. II. *Ibid.*, **5**, 165-178.

Roy, S. K. (1940). Certain inconsistencies in the mathematical theory of a new relativity of Dr. Sir Shah Sulaiman. *Proc. Nat. Acad. Sci. India*, A, **10**, 1-13.

Sulaiman, Shah (1940*a*). Modification of a relativity postulate. *Phil. Mag.*, (7), **30**, 49-54.

——— (1940*b*). Reply to the preceding paper. (Roy, 1940). *Proc. Nat. Acad. Sci. India*, A, **10**, 14-18.

Shanmugadhasan, S. (1946). On Mathisson's variational equations in relativistic dynamics. *Proc. Camb. phil. Soc.*, **42**, 54-61.

Vaidya, P. C. (1943). The internal field of a radiating star in general relativity. *Curr. Sci.*, **12**, 183.

——— (1945). Spherically symmetric line elements used in general relativity. *J. Univ. Bomb.* (N.S.), **14**, (3), 4-6.

THEORY OF SERIES

F. C. Auluck has written a note on some theorems of Ramanujan (1940). Haridas Bagchi has discussed a class of finite Riemannian integrals (1940). B. N. Bose has considered certain transformations in generalised hypergeometric series (1944). S. K. Basu has studied the total relative strength of the Riesz and Cesàro methods (1948*a*, 1949*a*). He has also written a note on total regularity of some integral and sequence transformations (1948*b*). In another paper he has discussed the total relative strength of the Holder and Cesàro method (1949*b*). K. Chandrasekharan has studied the summability of various series in a number of papers (1942*a*, *b*; 1943 *a*, *b*). K. S. K. Iyengar has obtained an equivalence theorem in a general field of summability (1942).

D. D. Kosambi has written a note on frequency distribution in series (1940). A. A. Krishnaswami Ayyangar has discussed the role of unit partial quotients in some continued fractions (1940). S. Minakshisundaram has developed a new summation process (1943). He has also written a note on the theory of infinite series (1941). In co-authorship of C. T. Rajagopal, he has made some investigations on a Tauberian theorem of K. Anand Rao (1946, 1947). In another note they have extended a Tauberian theorem of L. J. Mordell (1948). C. Racine has written a note on Frullani integrals (1947).

C. T. Rajagopal has written a note on the rearrangement of conditionally convergent series (1941*a*). In another note he has dealt with Abel's divergence test for series of positive terms (1940). He has written a series of papers on summability processes (1944, 1946*a*, *b*, *c*; 1948*a*; 1947*a*, *b*, *d*; 1948*c*, *d*; 1949).

Bibliography

Auluck, F. C. (1940). On some theorems of Ramanujan. *Proc. Indian Acad. Sci.*, A, **11**, 376-378.

Bagchi, Haridas (1940). Note of a class of finite Riemannian integrals. *Bull. Calcutta math. Soc.*, **41**, 103-112.

Bose, B. N. (1944). On certain transformations in generalised hypergeometric series. *Bull. Calcutta math. Soc.*, **36**, 74-79.

Basu, K. (1948). Note on some theorems on the Hölder and Cesàro means. *Bull. Calcutta math. Soc.*, **40**, 129-134.

Basu, S. K. (1948*a*). On the total relative strength of the Riesz and Hölder methods. *Bull. Calcutta math. Soc.*, **40**, 153-162.

——— (1948*b*). On the total regularity of some integral and sequence transformations. *J. Lond. math. Soc.*, **23**, 300-309.

——— S. K. (1949*a*). On the total relative strength of the Riesz and Cesàro methods. *J. Lond. math. Soc.*, **24**, 51-59.

——— (1949*b*). On the total relative strength of the Hölder and Cesàro methods. *Proc. Lond. math. Soc.*, (2), **50**, 447-462.

Chandrasekhar, K. (1942*a*). The second theorem of consistency for absolutely summable series. *J. India math. Soc.* (N.S.), **6**, 168-180.

——— (1942*b*). The absolute Bessel summability of series. *Bull. Calcutta math. Soc.*, **34**, 187-196.

——— (1943*a*). Bessel summation of series. *Proc. Indian Acad. Sci.*, A, **17**, 219-229.

——— (1943*b*). Bessel summability of the product of two series. *J. Indian math. Soc.*, (N.S.), **7**, 31-35.

Chatterjee, Anunoy (1948). On a continued fraction of a general type. *Bull. Calcutta math. Soc.*, **40**, 69-75.

Chowla, S., and Auluck, F. C. (1940). An approximation connected with exp. x. *Math. Stud.*, **8**, 75-77.

Iyenger, K. S. K. (1942). Notes on summability, I. An equivalence theorem in a general field of summability. *J. Mysore Univ.*, *B*, **3**, 123-129.

Kosambi, D. D. (1940). A note on frequency distribution in series. *Math. Stud.*, **8**, 151-155.

Krishnaswami Ayyanger, A. A. (1940). The role of unit partial quotients in some continued fractions. *Math. Stud.*, **8**, 159-166.
Levi, F. W. (1946). Rearrangement of convergent series. *Duke, math. J.*, **13**, 579-585.
Minakshi Sundaram, S. (1941). A note on the theory of infinite series. *Math. Stud.*, **9**, 78-81.
——— (1943). A new summation process. *Math. Stud.*, **11**, 21-27.
Minakshi Sundaram, S. and Rajgopal, C. T. (1946). On a Tauberian theorem of K. Anand Rao. *Quart. J. Math.*, **17**, 153-161.
——— (1947). Postscript to a Tauberian theorem. *Ibid.*, ser, **18**, 193-196.
——— (1948). An extension of a Tauberian theorem of L. J. Mordell. *Proc. Lond. math. Soc.*, (2), 50, 242-255.
Nigam, Tapeshwari Prasad (1940*a*). On γ—transformations of series. *Proc. Edinb. math. Soc.*, (2), **6**, 123-127.
——— (1940*b*). Summability of multiple series. *Proc. Lond. math. Soc.*, (2) **46**, 249-269.
Puri, Amritsaggar (1945). An identity and some deductions. *Math. Stud.*, **13**, 41-42.
Racine, C. (1947). On Frullani integrals. *J. Indian math. Soc.* (N.S.), **11**, 95-97.
Rajagopal, C. T. (1940). On Abel's divergence test for series of positive terms. *Math. Stud.*, **8**, 118-123.
——— (1941*a*). On the rearrangement of conditionally convergent series. *Ann. Math.*, (2) **42**, 604-613.
——— (1941*b*). Postscript to convergence theorems for series of positive terms. *J. Indian math. Soc.*, (N.S.), **5**, 113-116.
——— (1941*c*). Remarks on some generalizations of Cauchy's condensation and integral tests. *Amer. Math. Mon.*, **48**, 180-185.
——— (1944). The Abel-Dini and allied theorems. *Amer. Math. Mon.*, **51**, 566-570.
——— (1946*a*). On the limit of oscillation of a function and its Cesàro means. *Proc. Edinb. math. Soc.*, (2), **7**, 162-167.
——— (1946*b*). On converse theorems of summability. *Math. Gaz.*, **30**, 272-276.
——— (1946*c*). A note on the oscillation of Riesz means of any order. *J. Lond. math. Soc.*, **21**, 275-282.
——— (1947*a*). Some theorems concerning Riesz's first mean. *Acad. Scobe Sci. Puss. Inst. Math.* **1**, 11-20.
——— (1947*b*). On Riesz summability and summability by Dirichlet's series. *Amer. J. Math.*, **69**, 371-378.
——— (1947*c*). On Riesz summability and summability by Dirichlet's series. *Amer. J. Math.*, **69**, 851-852.
——— (1947*d*). Cesàro summability of a class of functions. *J. Indian math. Soc.* (N.S.), **11**, 22-27.
——— (1948*a*). On an absolute constant in the theory of Tauberian series. *Proc. Indian Acad. Sci.*, A, **28**, 537-544.
——— (1948*b*). Some limit theorems. *Amer. J. Math.*, **70**, 157-166.
——— (1948*c*). A series associated with Dirichlet's series. *Acta. Unv. Szeged.* Sect. Sci, Math., **11**, 201-206.
——— (1948*d*). On some extensions of Ananda Rao's converse of Abel's theorem. *J. Lond. math. Soc.*, **23**, 38-44.
——— (1949). On a Tauberian theorem of G. Ricci. *Proc. Edinb. math. Soc.*, (2), **8**, 143-146.
Sengupta, H. M. (1947). A note on quasi-monotone series. *Math. Stud.* **15**, 19-24.
——— (1950). On rearrangements of series. *Proc. Amer. math .Soc.*, **1**, 71-75.
S. Singh, Daljit (1945). On the series ΣS^r. *Math. Stud.* **13**, 59-60.
Upadhyay, S. D., and Das Gupta, P. N. (1947). On generalised continued fraction. *Bull. Calcutta math. Soc.*, **39**, 65-70.

TOPOLOGY

S. B. Krishna Murti (1940) gives a set of axioms for topology and proves their equivalence to those of Kuratowski under a certain restriction. V. S. Krishnan (1946) has studied the theory of weak homeomorphism between topological spaces and a characterization of completely regular spaces. A. S. N. Murti (1949) has proved some properties of a simply ordered space. A Ramanathan (1947*a*) has written a number of papers on Hausdorff spaces. He has shown that for Hausdorff topology of a set, the properties of bicompactness and those of having no stronger Hausdorff topology are not equivalent (1947*a*). In another paper he has proved the coincidence of the maximal-Hausdorff with the semi-regular H-closed spaces (1947*b*). He has also made a study of minimal-Hausdorff spaces (1948). T. K. Srinivasan (1947) has investigated some properties of distance functions. R. Vaidyanathaswamy (1942) has considered the characterization of the topology of a space, in terms of the lattice of its open sets (1942). He has also studied the theory of localisation in set topology (1944). He has also written a treatise on set topology, part I of which is based on lectures given by him at the University of Madras (1947).

T. Vijay Raghavan (1947) has formulated some simple examples of a connected linear ordered topological space which has the power of the continuum but which does not admit an everywhere dense enumerable subject and of a connected topological space which splits into exactly two connected open and closed sub-spaces when any one point is removed but is not a linearly ordered space.

Bibliography

Krishna Murti, S. B. (1940). A set of axioms for topological algebra. *J. Indian math. Soc.* (N.S.), **4**, 116-119.

Krishnan, V. S. (1946). A weak homeomorphism between topological spaces and a characterization of complete regular spaces. *Ibid.* **10**, 37-56.

Murti, A. S. N. (1949). Simply ordered spaces. *Ibid.*, **13**, 152-158.

Ramnathan, A. (1947*a*). Maximal—Housdorff spaces. *Proc. Indian Acad. Sci* A, **26**, 31-42.

——— (1947*b*). A characterization of maximal Hausdorff spaces. *J. Indian math. Soc.* (N.S.), **11**, 73-80.

——— (1948). Study of minimal bicompact spaces, *Ibid.*, **12**, 40-46.

——— (1949). On the strong extension of a T_1—space into a T_1—bicompact space. *Ibid.*, **13**, 25-30.

Sreenivasan, T. K. (1947). Some properties of distance functions *Ibid.*, **11**, 38-43.

Vaidyanathaswamy, R. (1942). On the lattice of open sets of a topological space. *Proc. Indian Acad. Sci.*, A, **16**, 379-386.

——— (1944). The localisation theory in set topology *Ibid.*, A, **20**, 51-61.

——— (1947). Treatise on set Topology. Part I. *Indian Math. Soc. Madras*, (Based on Lectures).

Vijaya Raghavan, T. (1947). On two problems relating to linear connected topological spaces. *J. Indian math. Soc.* (N.S.), **11**, 28-30.

PART II: GEODESY 1939-1950

by B. L. GULATEE, M.A. (Cantab), F.R.I.C.S., M.I.S. (Ind)
Director, Geodetic and Training Circle, Survey of India

INTRODUCTION

Geodesy literally means 'Division of the Earth' (Greek *gē* the earth; *daiō*, to divide), and in the early days used to be regarded as that branch of applied mathematics devoted only to the shape and size of the earth and the fixation of exact positions of points on it. The operations necessary to achieve this end are:—

1. Principal triangulation and base measurements.
2. Measurements of elevations by trigonometrical and spirit levelling methods.
3. Geodetic astronomy i.e., determinations of astronomical latitudes, longitudes and azimuths. These provide the direction of gravity at a place.
4. Observations of the force of gravity.

The earth's surface is very irregular and cannot be expressed by a simple mathematical formula. It is neither possible to carry out any geodetic calculations on it nor to speak of its dimensions in precise terms. Mean level of open sea is much more regular and if carried inland through imaginary frictionless channels, provides a level surface of the earth, called the geoid. Although this surface is much more tractable than the actual earth, it is still too complicated to be determined by a single mathematical expression. It is, therefore, usually depicted with reference to an oblate spheroid, whose dimensions are called the figure of the Earth. This figure is characterized by two elements—a the radius of the equator and ϵ the flattening.

The earlier method of determining the shape and size of the earth was to determine astronomically the angular distance between a number of points and also measuring the linear distance between them. Newton was the first to show from his law of universal attraction that the earth should be an oblate spheroid, the flattening being due to its axial rotation. This implies that the length of a degree of latitude should increase from the equator to the poles. The classical attempts to test Newton's view regarding the figure of the earth form fascinating reading. Towards the end of seventeenth century, geodetic measurements of meridian arcs in France revealed that the length of a degree of latitude was shorter in northern than in southern France, which would be the case if the spheroid were prolate. This aroused acute controversy and in 1735 the French Academy of Sciences organized two expeditions—one to measure a meridian arc in the neighbourhood of the equator and the other near the pole. The first party under Bouguer & de La Condamine took about 9 years to measure an arc of 3° near Peru, and the second under Maupertius measured an arc of 57′ in eighteen months in Lapland. Maupertius measured his base line on a frozen river and later considerations show that he got an oblate spheroid by sheer good luck as his measurements were not accurate. In any case, he gave out his results without waiting for the other expedition and received a splendid ovation. The so-called 'earth-elongators' were quietened for ever and Maupertius was painted in a splendid costume flattening a globe with a superb gesture.

In the following years, many great arcs were measured to get more knowledge on the subject. Mention might be made of the work of the Abbe' de la Caille, a renowned astronomer who was sent to Africa by the French Academy in 1751.

His magnum opus was the measurement of the arc of the meridian near Cape Town to settle the question whether the southern half of the globe had the same shape as the northern. He was completely baffled by his results which appeared to support the hypothesis that the earth was a prolate spheroid. It was not till 1836 that the puzzle was solved by Sir Thomas Maclear (1866), Astronomer Royal at the Cape of Good Hope. One of La Caille's base stations was near a mountain and its deflection vitiated his observed astronomical value.

In India the figure of the Earth was derived by Everest (1879) in 1825 from arc measurements involving triangulation and astronomical determinations. He had at his disposal, the data of seven arcs of amplitudes ranging from 1°:37′ to 15°:57′. Combining them in pairs, he found that the resulting values of semi-major axis and ellipticity of the spheroid varied considerable. Values of ϵ ranged from 1/300.8 to 1/416.5 and those of a displayed a range of 6 miles. He realized correctly that these discrepancies were due to tilting of the plumb-line produced by visible as well as buried topography, which vitiates the astronomical values of the terminal latitudes. He chose the two longest arcs, one in India from Pannae to Kalianpur of amplitude 16° and one in Europe from Formentera to Greenwich of amplitude 13°, for his final deduction. The constants so derived have been in use from 1830 to the present time in the Survey of India for all calculations of survey into which the elements of earth's figure enter. Everest's figure of the Earth differs much from that derived from modern data but it has not been found practicable yet to abandon it.

Pendulum observations for measuring the force of gravity were first initiated in India between 1865 and 1873 when two Kater's invariable pendulums made of brass with steel knife edges were lent to India by the Royal Society. Basevi and Heaviside (1879) took observations with them at 31 stations, the results created quite a stir at the time amongst the leading geodesists of the world (Helmert), 1890), and presented great difficulties of interpretation. Helmert opined at the International Geodetic conference of 1900 that they were suspect, as certain essential corrections were not applied.

After 1874, no pendulum observations were taken in India but the plumb-line deflections for the direction of gravity continued to be observed in different parts of the country and by 1900, a considerable body of data had accumulated. It was found that the bulk of the deflections did not follow the visible topography. As an example, throughout the extent of the Gangetic plains, the plumb-line was deflected away from the Himalaya Mountains. To explain these Burrard (1901) postulated the existence of a subterranean line of high density which he called the "Hidden Range" extending from Baluchistan to Bengal and Orissa.

The observations brought to light yet another important geophysical phenomenon namely that the mountains do not deflect the direction of gravity to the extent warranted by their mass.

It was considered of the greatest importance to obtain an independent confirmation of these phenomena by associating gravity determinations with deflections of the plumb-line. Von Sterneck's pendulum apparatus was purchased in 1902 at the advice of Helmert and since then except when prevented by the world wars, pendulum observations have featured regularly in Survey of India activity. Lately these have been supplemented by the Frost gravimeter.

Both the direction and intensity of gravity are affected by visible and invisible anomalies of crustal density. Changes in them give an indication of underground anomalies of density of which there may be no other surface indication. We thus see how gravity and astronomical observations which were first intended for purely geodetic purpose of determining dimensions and shape of the earth gave important clues for the service of structural geology. Different measured arcs produced

different values of the elements of the earth and it was soon realised that the distribution of densities in the outer part of the crust was mainly responsible for these deviations.

In about 1850 Pratt and Airy working on the deflection data in India concluded that the mountain systems were underlain by deficient density which reduced their effective attraction. From these studies emerged the Theory of Isostasy by Hutton in 1880 and discussions on this subject have forced the geodesists into broader fields covering geology, seismology, volcanology, and oceanography. This has paved the way for geodesy to overlap certain branches of geophysics and to assist in such important practical operations as prospecting for oil and minerals.

A comparison of the astronomical and geodetic positions of survey points gives a powerful tool for investigating the degree of isostatic compensation and its nature. The astronomical observations of latitude and longitude initiated for determining the local and general figure of the earth were found useful and necessary for testing such hypotheses as Continental drift, Variation of Latitude and changes in the earth's crust wrought by earthquakes. Precise re-levelling has been utilised to give very useful information about the slow tilting of the earth's surface. Geodetic observations and reductions have thus come to occupy a prominent place in the earth sciences and items such as Magnetic and seismic surveys and their applications to geophysical prospecting; Latitude variation; Tidal analysis and mean sea level can now legitimately be included in geodesy. The problems associated with M.S.L. provide a common meeting ground for geologists, geodesists, meteorologists and hydrographers.

Of late years the problem of finding the figure of the earth has assumed a much more comprehensive nature than its original formulation. It has been realised that the measurement of detached arcs of meridian and parallel can not give a representative picture of the earth as a whole. The two modern methods for tackling this problem are: (a) Astronomical Levelling to delineate the geoid in detail. This involves a lot of work, astronomical stations being required at intervals of ten to fifteen miles along closely spaced sections on all the continents. (b) By gravity field. This again requires a net of gravity observations uniformly distributed over the entire globe.

The main advance in geodesy lies in the furtherance of these observations in different countries by the use of quicker and more precise instruments. These observations are also indispensable for investigations of the nature of equilibrium in the upper layers of the earth.

A recent development brought about by the impact of World War II is the application of Radar to Geodesy and to Survey. The European countries have made a considerable progress in late years in developing radar devices each characterized by its own code name such as Gee, Oboe, Shoran, Loran, etc. The application of these methods to measurement of geodetic distances as also to the determination of the geographical positions on the earth in relation to accurately established ground stations is of considerable importance. The outfit requires is rather elaborate and India has made no start yet with this new and important aspect of geodesy.

The study of geodesy in India has been mainly confined to the Survey of India. Lately certain contributions have been made by the Oil Companies making use of their gravity data. The Universities have only just started to take interest in geophysics and some elementary courses in geodesy might get started in the near future..

II. *Triangulation and Base-measurement.*

In 1938-39 financial stringency severely restricted the geodetic field programme and no geodetic triangulation was carried out. War broke out in 1939

and all geodetic activities remained practically in abeyance till 1947. The war, however, helped in achieving certain objectives, which would have presented practically insuperable difficulties during peace time. The Survey companies of the Allied forces effected several important addition, notably Malaya-Siam, and India-Persia-Iraq-Syria triangulation connection. A continuous chain of triangulation now exists from Syria to Malaya. Its extension to Australia in the East and to European triangulation in the west is a problem of consideratble interest to the geodesists.

Apart from errors of observation, the actual discrepancies at the junctions of these triangulations are due to different apheroids and datums used by the various countries. B. L. Gulatee (1947*a*) has made a full study of the data and the discrepancies as also of the gaps that need filling in. The gap between the triangulation systems of Malaya and that of N. E. I. on the main land of Sumatra or Java is too wide to be bridged by ordinary triangulation methods. A suitable radar technique or parachute flare method will have to be adopted. A similar method will also be necessary to connect Australia with Timor and other neighbouring islands.

J. de Graaff Hunter and B. L. Gulatee (1946) have examined the precision of the trans-Persian triangulation (1941-44) linking Iraq and India. This triangulation can be broadly divided into two main portion. (a) the net work in N.W. Persia reaching as far as Isfahan consisting mostly of elongated braced quadrilaterals and having a precision of about 1/15,000, and (b) a chain of good quality triangulation from Nain to India which can be made to achieve an accuracy of 1/50,000 if scale is toned up by a little additional work. The discrepancies between the Persian work in Iraq origin terms on Clarke 1880 spheroid and published Indian data on Everest spheroid with Kalianpur as datum are $+12''.1$ in latitude and $-4.4''$ in longitude. This represents the discrepancy that would exist between the maps of India and Persia at their functions.

Geodetic triangulation is required to provide a rigid framework for the control of topographical and cadastral surveys. In India, a skeleton framework was executed in the last century consisting of a series of chains about 100 to 200 miles apart (Anon., 1950). A good deal of it is of secondary quality. This and the topographical triangulation based on it, although adequate for providing a framework for the one-inch topographical map of India, are quite insufficient to meet the needs of large scale cadastral, hydro-electric, irrigation and other development schemes. There are numerous urgent demands now-a-days on the Survey of India for large scale maps. One of these was in the Kathiawar area for the development of the Port of Kandla. To provide the requisite planimetric control, a geodetic base $5\frac{3}{4}$ miles long was measured in 1949 and the old secondary triangulation was reobserved for a length of about 100 miles with modern instruments. Laplace observations for control of aximuth were made at two stations. (Anon., 1950*b*).

A systematic programme of re-observation of the entire secondary triangulation in India extending over a period of years is envisaged. It is programmed in 1951 to establish a new astronomical datum, measure a geodetic base and execute a series of precise triangulation in the Andaman Islands and also to observe a new series of geodetic triangulation for the demarcation of the East-West Bengal Boundary.

No uniformity exists in various countries regarding the standards of length in terms of which the lengths of their bases are measured. In 1830, Everest (1870) brought to India a ten foot standard bar A made of steel after comparing it against British standards in England. In terms of this bar, ten primary base lines were measured between 1832 and 1869 with considerable accuracy. This

bar was of defective design and at the beginning of this century, the Survey of India acquired the following modern metric standards of one meter length:—

1—metre Nickel Bar
1—metre Fused Silica Bar
1—metre Nickel Steel & 1-metre Invar Bars.

Of these, 1-metre Nickel bar which was made by Societe' Genevoise d'Instruments de Physique in 1911 is intended to be the fundamental standard and the others serve as auxiliaries for check and working purposes. In view of the fact that all material bars no matter how carefully handled undergo gradual variations with time, it is essential that the working bars should be intercompared frequently and that the standard should be compared against European standards after some years. The Nickel metre was standardized at the National Physical Laboratory, Teddington in 1914. It was sent back to N. P. L. in 1930 for re-standardisation and was found to have shortened by $4/10^6$. This is considerable and can only be attributed to some unrecorded accident. The bar was again standardized at the National Physical Laboratory in 1947 and during the period 1931-47 was found to have decreased in length by $1/0{\cdot}4 \times 10^6$, which is very satisfactory.

Gulatee (1946*a*) has reviewed the disadvantages and confusion resulting from a lack of uniformity of length standards all the world over. There is at present a considerable diversity in the foot-metre ratio and an inter-comparison of national standards of all countries is most desirable. Steps to this effect are being taken by the International Union of Geodesy and Geophysics and it is proposed to establish one or more International base-lines in each continent .

In geodetic work, a 'Laplace station' connotes a point where both longitude and azimuth have been observed astronomically. Laplace stations are necessary in any primary triangulation to control the error accumulated in geodetic azimuth. In the Indian geodetic triangulation, 72 Laplace stations have been observed in all. Hunter (1947) has considered their value in controlling the azimuth of a precise traverse.

During the war, certain projections were introduced for the use of the military. The normal spherical computation of triangulation is difficult and is a specialist's job. These projections were so designed that the military could work their ranges and bearings in rectangular terms, as if the earth were flat and apply certain corrections for deformations in scale & bearing. The Lambert Orthomorphic conical projection was employed in several theatres of war, but great difficulty was experienced in finding a suitable formula for angular corrections on this projection. Gulatee (1946*b*) has derived a formula involving spherical coordinates and has discussed why the deformations due to this projection cannot be expressed in terms of rectangular coordinates by straightforward direct formulae. He has pointed out the superiority of the Decumanal Mercator & Transverse Mercator Projections over the Lambert Orthmorphic in this respect.

III. *Spirit-Levelled and Trigonometrical Heights.*

The first levels in India were started as early as 1858 and a skeleton precision net of a linear extent of 18000 miles was completed and simultaneously adjusted by the method of least squares in 1909 (Anon., 1910). This forms the fundamental framework for the whole country and the tertiary levelling which rests on it via the intermediary of the secondary levelling provides the bulk of the data for preparing accurate large scale maps. These enable the engineers to proceed with their projects and planning operations.

Improvements in technique and knowledge led the International Geodetic Conference in 1912 to sponsor a new category of levelling called the 'Levelling of High Precision'. India made a start with it in 1914 and out of an estimated

total of 15,800 miles for the length of the new High Precision Level net, only 10,023 miles had been carried out by 1938. During the two field seasons 1939-41, 706 miles of high precision levelling in one direction equivalent to 353 miles of completed levelling was carried out. No work was carried out from 1941 to 1946. From 1946 to 1950, about 1400 miles of levelling were added, bringing the completed total to 11781 miles.

An important achievement has been the connection of the Indo-Burma levelling to that of Siamese levelling by No. 2 Indian field Survey Company in December 1946. The discrepancy at the junction between the two systems of levelling is 1.567 feet. The Burma levelling is based on the Amherst Tidal Observatory as datum and the datum of the Siamese levelling is the Tiday Observatory at Koh Hlak. A direct connection by levelling of the two datum is desirable.

In general, the secondary and tertiary levelling are done by the Survey of India for extra departmental agencies as paid for work. The progress of levelling in India has been slow as the country was comparatively undeveloped. The situation has altered considerably now and levels are greatly in demand on account of important developments in the country. As a long term policy the objective should be to cover the whole country with a net-work of level lines, so that there is no place which is more than say 20 miles from a stable levelling bench-mark. Gulatee (1949*a*) has made a case for acceleration of levelling programme, calling for active cooperation of the local governments, P.W.D., Railway and other outside agencies.

Due to the sparseness of spirit levelling, the bulk of the height control in India, is provided by trigonometrical heights obtained by the observations of vertical angles. These are not so precise on account of the uncertainty introduced by terrestrial refraction. In particular in the plains, where grazing rays are inevitable, spirit levelled connections have revealed errors of as much as 31 feet in these heights. Chart VI, Chapter I of Survey of India Technical Report 1948-49, Part III shows the estimated maximum discrepancy between trigonometrical and spirit levelled heights of stations of geodetic triangulation. It would be seen that there are vast areas in which triangulated heights are estimated to be in error by more than 10 feet.

An extreme case is the determination of altitudes of distant high peaks fixed by observations from the plains where refraction effect can amount to several hundreds of feet and its estimation can be in error by as much as 25%. Mount Everest presents notable example. The significance of its adopted height of 29002 feet is not generally understood and quite a number of other heights have been quoted for it and have been put on the maps which makes for confusion. In a paper on the subject, Gulatee (1950*a*) has explained the outstanding difficulties associated with high Himalayan peaks. The determination of their height is a problem of higher geodesy involving a knowledge of advanced theory of refraction, plumb line deflections, gravity, geoids, datums of reference and so on. Observations to Mount Everest were made in 1849-50 and judged by modern standards, both the observations and computations were faulty. The adopted figure of 29002 feet may be in error by as much as 100 feet but it is not possible to finalize it as the existing observational data is far too incomplete and many doubtful factors enter into it. A scheme is outlined in this paper for fresh observations to be carried out on systematic lines, which will put an end to the controversy.

There are several important scientific and practical problems associated with levelling. It can be used to test the changes of levels in unstable regions where slow upwarping or downwarping of the crust, may be in progress. There is a general belief that certain areas in India such as the Kathiawar Coast, the land

near the Rann of Kutch and the alluvial areas of south Bengal are sinking. In Particular the stability of Calcutta has, since some time past, been the subject of grave concern. The question has been examined in the light of available tidal and levelling data (Gulatee, 1948-49 *a*). The results indicate that apart from local sinkages at places, there is no conclusive evidence of a general downwarping of the crust in South Bengal. The city of Calcutta and the region round it appear to have remained more or less stable.

The Siwālik range is of recent origin and is believed to be rising. A line of precision levelling from Roorkee to Hardwar originally levelled in 1908 was relevelled in 1948. It passes through the gap in the Siwalik range carved by the river Ganges. The results indicate an upwarping of the Siwalik axis at the rate of about 1 inch in 40 years. (Gulatee, 1948-49*b*).

IV *Deviation of the Vertical*

The study of deflections in India on some sort of a systematic basis was started by Burrard (1901) in the beginning of this century. In those days the data was sparse and the plumb line deflections were plotted and shown vectorially by arrows. Certain important characteristics about their distribution were noticed such as their being deflected away from the Himalayas in central India and pointing towards a hidden line of apparently excess density in the plains. As more and more data accumulated, it was considered that to make a detailed study of the hidden mass anomalies in the earth's crust it was much more conveint to draw contours of the geoid with respect to a chosen reference spheroid by integrating the plumb line deflections along meridians and parallels. This geoid is an equipotential surface of the earth's gravitational and rotational field and coincides generally with the mean sea level in the open ocean. Incidentally, this surface also provides a powerful tool for the determination of the earth's figure. An excellent attempt in this respect is that of Hayford. He derived the Dimensions of the best fitting spheroid to the compensated geoid in U.S.A. (Hayford, 1909) and this figure was accepted by the International Union of Geodesy and Geophysics in 1925 as the International Sphreoid.

In India, the total number of stations at which the deviation of the vertical has been observed up to the year 1950 is 1210. The results are printed in the Survey of India annual Geodetic and Technical Reports. Charts. XXII & XXIII of Technical Report, Part III, 1950 show the Geoid and the Compensated geoid with respect to the International spheroid. These charts have been drawn by utilizing the above mentioned deflection stations, the majority of which are sited at an average distance of about 15 miles on two main lines—one running east to west from Burma to Persian Border and the other from north to south through Cape Comorin.

The geoid in India presents some very remarkable features. One of its most salient characteristics is, that its undulations are independent of the major visible features, indicating that there are equally important hidden features at work. The 'Hidden Range' traversing the extent of Central India postulated by Burrard is confirmed and brought vividly to the eye. Another very disturbed region is that of South Burma and Malaya. While the geoid in peninsular India displays humps of the order of 40 feet or so above the International spheroid, its rise from Mandalay to Victoria Point is 110 feet in a distance of about 1000 miles which is phenomenal and without parallel anywhere else in the world. Even the extensive Hidden Range pales into insignificance as compared to it. Observations of plumb-line deflections in Siam, Malaya and Dutch East Indies are very desirable to see if the high rise continues in these areas.

From Chart XXIV, Survey of India Technical Report 1950, Part III it would be seen that the main geoidal framework in India is pretty well braced up except

in the north, where observation of the section Jalpaiguri-Pota-Meerut along the North -East Longitudinal series is indicated. Some further stations in Burma along Mandalay to Dibrugarh or on Manipur road are desirable to extend the geoid into unexplored regions.

For different studies, the geoid has to be modified in certain respects. The definitions of the various geoids in use in India and the data on which they are based is given in Survey of India Geodetic Report (Gulatee, 1948-49 c). The geoidal charts in India have provided a broad framework for the study of deep seated curiosities in the earth's crust well below the limit of geophysical prospecting. The next step is to narrow down this framework by putting in more deflection stations to gain further detailed knowledge of superficial effects.

V. *Intensity of Gravity.*

A programme of pendulum observations in India had been started in the beginning of this century and by the end of 1939, a total of 564 stations were observed, distributed more or less uniformly all over India and Burma. Gravity anomalies have been derived from these observations (see Charts VI and VII, Geodetic Report, 1940) and have confirmed to a large extent the conclusions derived from geoids. Their intensive study has revealed considerable areas in India where isostasy is not fulfilled. There is first the so-called 'Hidden Range', a subterranean chain of rocks crossing penisular India from Baluchistan to Bengal. Associated with the Hidden Range are the Himalayas and the Indo-Gangetic trough both presumably owing their origin to the same tectonic cause. Gravity data indicate, that the Gangetic plain is an area of underload, deficiency there being equivalent to a skin density of -500 to -2000 feet of normal rock condensed on to sea level surface.

Just as the deflections in India are independent of topography so are the gravity anomalies independent of local geology in several important regions. As examples might be mentioned the negative anomalies over the high density Deccan Traps and the broad extensive positive belt of the Hidden Range in peninsular India.

Gravity observations on oceans are no less important than on land but are much more troublesome as the pendulums have to be swung in a submerged submarine to get over the shaking and accurate position finding is more difficult. Gravity observations in the waters of Dutch East Indies by V. Meinesz (1923-32) have brought to light features of unique importance. In this connection, the pronounced negative anomaly zone in the Indo-Gangetic sedimentary area at the foot of the Himalayas is of special interest. V. Meinesz found a narrow strip of strong negative anomalies running parallel to the west Coast of Sumatra. Recent pendulum observations in the Andaman Islands and South Burma indicate that this negative strip passes through the Nicobar and Andaman Islands and then links up with the line of negative trough through W. Burma to the above mentioned negative trough which forms a fore deep of the Himalayas and then possibly continues on to Russian Turkistan. This line is obviously a tectonic line of earth's structure and marks a region of structural instability.

There are important gaps regarding gravity observations in such areas as the Bay of Bengal, Indian Ocean and South China Sea, which when filled in will yield results of capital importance. The indications are that gravity anomalies in these regions will be predominantly positive.

In 1854, Pratt published a paper in the Phil. Trans. of the Royal Society in which he calculated the plumb-line deflections due to Himalayas at three stations. He found these deflections to be greater in amount than their observed values. This led him to formulate his theory of compensation (Pratt, 1859, 1855) namely that the irregularities of mountain surfaces have arisen from the vertical

expansion of the earth's crust from depths below. In this way the surface features get underlain by masses of deficient density. Airy (1855) propounded a rival hypothesis that mountains and plateaus have roots below them penetrating into the denser substratum, the whole block floating in hydrostatic equilibrium. Considerable amount of work has been done in reducing the observed gravity values on both these systems to see which gives a better accord with facts.

Seismological evidence in recent years, has, however shown that the normal structure of the earth's crust is by no means homogenous, but consists of three layers possessing different properties. The interfaces of these layers are about 10 k.m. and 30 k.m. below sea-level, the discontinuity of density at these layers being about 0.2 and 0.5 gm/cm^3 respectively. Modern concept of compensation is quite different and takes the form of elevation or depression of the bases of the sedimentary, granitic and basaltic layers above or below their normal levels.

Much research has been carried out by the Survey of India on a qualitative basis to bring out a relation between gravity anomalies and crustal structure lines. Glennie (1932) has put forward the crustal warp theory to explain the tectonic features of India. Negative anomalies are assumed to be due to a downwarp and positive ones to an upwarp in the normal arrangement of the three layers compirising the earth's crust. A quantitative delineation of the warps at the interfaces is, however, not capable of a unique solution, is rather laborious and is a task for the future.

Glennie has also suggested that under the Gangetic Plain is the southern margin of the great geo-syncline which formed the basis of the Tethys. The formation of this geo-syncline involves a deep-seated down-warping of the earth's crust and the Hidden Range marks the line along which the balancing uprise took place.

Observed values of gravity appertain to different level surfaces of the earth and before any use can be made of them, they have to be reduced to the same surface usually the geoid. In this process, uncertainties are introduced due to various causes and considerable experience is required for the choice of reduction to be used for a particular purpose. With the advent of gravimeters, very precise gravity material is accumulating fast and it is imperative that for problems requiring global gravity data, the reduction methods employed by various countries should be on a uniform system. Considerable attention has been given lately to ensure this. In India, H. J. Couchman (1915) produced tables for isostatic reduction on Hayford's hypothesis. He, however, adopted different values for the radii of inner zones to those chosen by Haford, Also slightly different assumptions were made as regards the number of compartments in the various zones, the depth of compensation and the gravitation constant. The pendulum stations in India have been reduced with the help of these tables, but their non-conformity to Hayford reduction table have produced an unnecessary complication. Investigation (Gulatee, 1949 *b*) has shown that due to this, the results are liable to an error of 3 mgals. In particular, the effect of outer zones 1-9 is burdened with a systematic error of ½ mgal, as the height estimations were made from very sketchy atlases. Corrective action has been taken in this respect to ensure uniformity with the reductions of other countries.

The Hayford gravity anomalies are computed on the assumption that the crustal rocks have a density of 2.67 and can only be regarded as preliminary due to the fact that vast areas in India are covered by light alluvium and by dense traps where the above density value does not hold. Geodesists had not paid much heed to local geology as in the vast areas that they deal with, things get averaged out. Evans and Compton (1946) have pointed out some of the consequences arising from lack of collaboration between geodesists and geologists in

this respect. They have considered the results at 6000 gravity stations established before the war by the Burma Oil Company in Eastern Bengal, Shan Plateau and Burma. For the interpretation of their results they applied the corrections for local geology upto a depth of above 8 miles. It was found that at some places, the anomalies were modified considerably by the application of geological correction. While density correction upto sea level can be made without much difficulty, correction for density below that surface necessarily involves many doubtful assumptions and much geological information has to be collected to get a reasonably accurate knowledge of the rock densities down to a considerable depth and to assess the effects of compaction there.

In a Professional Paper on gravity anomalies and the figure of the earth, Gulatee (1940) has considered the fundamental problems of higher gravity. He has discussed the various theories of compensation and their applicability to the determination of the disturbing masses in the earth's interior. None of these pictures of compensation can claim to be true in any absolute sense, but they provide a convenient reference system from which to reckon the departures. A study of the mountainous regions in particular reveals that the departures in nature from any form of isostasy are much greater than the differences between the various systems.

The use of gravity anomalies for the determination of the flattening of the earth and the problem of linking of gravity and deflection data for the choice of a reference spheroid for geodetic work are also discussed in this paper.

The determination of local humps of geoid over a reference spheroid requires a knowledge of gravity all over the globe. This is a matter for international co-operation and the desideratum for this gravity observations over oceans and in southern hemisphere.

Another geodetic reason for making dense gravity net especially round the origin of a Survey is to find absolute deflections at the datum. The deflections determined so far are by astronomical levelling (which is triangulation combined with astronomical determinations of latitude, longitude and azimuth). As triangulation connections between various countries are beset with several difficulties, the observed deflections are generally not directly comparable with each other. Gravity anomalies provide absolute deflections and a great advance will have been made when a sufficient number of these uniformly distributed all over the globe became available.

The observations with pendulums are of a cumbersome nature and a density of one station in an area of about 4000 square miles as was achieved by the end of 1939 was about the limit of economy to which they could be put. When they were initiated, the main idea was to obtain confirmatory evidence of certain conclusions drawn from plumb line deflections. As has been seen above they have served this purpose admirably.

As ideas matured, it was realised that a much closer mesh was needed for the solution of several important problems such as the determination of local undulations of the geoid and the finding of absolute deflections at the datum. It was considered that a 10-mile grid of gravity stations as against the existing 70 mile grid of pendulum stations would be useful for the above problems and would also give clues as to curiosities within the first ten miles of the earth's crust for intensive study. This was approved by the Geophysical Planning Committee set up by the Government of India in 1946. The project involves some 30,000 stations and would be impracticable with the slow pendulum technique.

It became apparent that if the objective was to be achieved within any reasonable period of time, quicker instruments such as the gravimeters would

have to be employed. Consequently a Frost Gravimeter was obtained in 1947 and is now in operation (Anon. 1947). This is a far more precise instrument than the pendulum apparatus, readings being possible to 0.01 mgals and a repeat accuracy of 0.1 mgal being comparatively easy.

In the season 1948-49, observations have been made at 101 new stations with the Frost Gravimeter in the Raniganj coalfields area and in the Nagpur area. The work in Nagpur area is still in progress.

Gulatee (1948-49, 1950 *a*) has analysed the results in the Raniganj coalfields area and some interesting features are brought to light. Gravity anomalies at the observed stations have been computed on seven different hypotheses. While the present spacing of stations cannot locate anything like the actual coal seams, it can help in structural investigations such as delineating the extension of the Raniganj Coal bearing series under the alluvium and in pointing out areas for more intensive study.

Thirty-six old pendulum stations were also re-observed in the same field season, and useful information gained about the precision of older work.

The national base station for gravity in India is Dehra Dun. No absolute determination were made here, but a connection with various European stations such as Kew, Potsdam, Genoa, etc., has been effected several times since 1904 with the help of comparative pendulum observations. These connections have been rather unsatisfactory as the various values displayed a range of as much as 25 mgals. The values of gravity at the reference stations of some other countries were also not well determined and in 1948 G. P. Woollard of the Oceanographic Institution, Woods Hole, Massachusets carried out a world girdling tour on a special military plane to effect this with a special long range geodetic gravimeter. In India, he was able to observe at Calcutta, Gaya, Allahabad, Kanpur and five stations in Delhi. The connection from Delhi to Dehra Dun was established with the Frost Gravimeter. The final value at Dehra Dun (Gulatee, 1948-49, 1950 *b*) obtained by Woollard and Gulatee from Washington via Delhi is 979.063 Cm/Sec^2.

VI. *Geophysical Prospecting.*

Although, strictly speaking, Geophysical Prospecting is not a branch of Geodesy, it is an allied subject and geodetic data can be of considerable help. A brief reference only will be made to some of the work carried out during the period under report.

The manganese deposits in India especially in the Central Provinces have always played an important role in the supply of ore to foreign steel producing countries. These ore-beds are blanketed over by alluvium in many places, and during World War II an investigation was carried out by Gulatee (1947 *b*) to study the applicability of geophysical methods of prospecting to the systematic location of these hidden bodies. A site was selected in the alluvial area of Parsoda in Nagpur district, in which the ore occurs in reef as well as in boulder form. The boulders are embedded in shallow alluvium and are scattered all over the area: the reef is more compact but on account of having been subjected to considerable folding, its thickness and width keep on changing all the time. From the point of view of testing the geophysical methods, the site selected was not ideal as the area contained rich boulders which contaminated the measured values. But the main consideration was that a company was opening up the area and it would be possible to verify the geophysical indications. A gravity method was utilized in the first instance as manganese has a distinctly higher density than its surrounding rocks. The instrument actually used was the Gradiometer which is a modification of the Torsion balance and measures only the horizontal gradients of gravity. Six traverses were run perpendicular to the conjectured

direction of the reef with stations about 40 feet apart, and the reef was clearly located, maximum gradient being 114 E. It was not realized till the party returned to headquarters and some samples were tested that the ore body in this area contained some minerals which are magnetic. Subsequently in 1947 the area was covered by two Watts Vertical Force Variometer and very clear indications of the reef were obtained, the maximum anomaly being 330$^{\gamma}$. It was clearly established that both magnetic and gravimetric methods are suited to the problem and the reef could be fairly accurately delineated.

Ramachandra Rao (1947) has carried out earth resistivity surveys in the Godavari river basin in connection with the Ramapadsagar project and had also presented a review of the verification of the electrical indications by drilling in a number of cases. He (Ramachandra Rao, 1949) has also given an account of recent advances in the design of special geophysical instruments and has pointed out the desirability of using them for geophysical exploration in India.

Gulatee (1947 *c*) has given an account of Geophysical work in India and has also indicated in broad outline some fruitful lines for further research.

Dessau (1947) has given a history of past geophysical work in India and has emphasized the necessity for a full appreciation of the limitation of Exploration Geophysics.

He (Dessau, 1948) has also described the work of the geophysical section of the Geological Survey of India from 1945-48.

VII. *Terrestrial magnetism.*

The magnetic survey of a country is usually undertaken by a geodetic organisation, although only some aspects of Terrestrial magnetism come under geodesy. The following account is confined only to the magnetic activities of the Geodetic Branch of the Survey of India.

The objects of a magnetic survey are

(*a*) The advancement of science. Geo-magnetic observations are essential for fundamental research on the elucidation of the magnetic & the electrical conditions of the earth and for obtaining information about the electrical conductivity of the earth's interior.

(*b*) The determination of magnetic variation which is of practical utility to cartographers, geologists, mining engineers and the army.

(*c*) The detection of magnetic ores and underground structure.

The immediate objectives in a magnetic survey are :—

(*a*) To determine the magnitude & direction of the magnetic force in all parts of the country at some specified epoch and to show it on appropriate charts.

(*b*) To determine the diurnal, annual & secular changes in this force. These apart from their own scientific use are indispensable for the fulfilment of (a).

For the above, observations of three kinds are needed (*i*) at field stations (*ii*) at permanent observations & (*iii*) at Repeat stations.

The magnetic survey of India was begun in 1901 on the recommendation of the Observatories Committee of the Royal Society. Five permanent observatories at Dehra Dun, Kodaikanal, Barrackpur, Toungoo & Colaba were selected as base stations. Observations with absolute instruments for declination, dip & horizontal force were carried out at 1400 field stations and 80 repeat stations, so distributed as to give a density of four field stations per degree sheet and one repeat station per 1/M sheet. The field work was completed in 1913, but the reduction of the huge mass of observational data to a particular epoch involved a large amount of computation and could not be taken up immediately as the war supervened and the officers were diverted to other duties. At the end of the war, the survey was brought up-to-date to the epoch 1920 by observations

at repeat stations and the results did not get into print till as late as 1925 (Gulatee, 1946*c*).

It would be serviceable to put on record here the later developments. At the close of the general magnetic survey, Barrackpur observatory was closed down and it was the intention to keep the other observatories in continuous operation and to observe the repeat stations at 5-yearly intervals. This would ensure reliable picture of secular variation pattern being obtained and would enable the survey to be brought upto date at any later time with observations at a limited number of field stations only.

Financial stringency in 1923 led to the abolition of the magnetic party and to the closing down of Toungoo and Kodaikanal Observatories. The Dehra Dun underground observatory was put out of commission in 1943 as it was flooded during the rains. The scheme of visiting the repeat stations at intervals of five years to keep track of the secular change of magnetic elements was not implemented on account of lack of finance.

The practical effect of the deterioration of magnetic knowledge is explained by Gulatee (1946*c*) in a paper in which he stresses the importance of reopening of the five permanent magnetic observatories in India and declaring the 80 magnetic Repeat stations of India as protected areas to guard against their encroachment by structures containing iron.

The Geophysical Planning Committee in 1949 has recommended strongly to the government that the magnetic observatory at Dehra Dun should be re-established as the loss of continuity in magnetic data means a serious loss to science. Financial stringency has so far stood in the way. The Kodaikanal Observatory which was closed in 1923 has been restarted in 1949 and Alibag observatory is running uninterrupted.

On the outbreak of World War II, demand for data of greater accuracy especially for maps for military purposes greatly increased. Observations at all repeat stations were carried out in the years 1943-45. A lot of extra-departmental information as regards magnetic declinations was collected and compiled for preparing the latest isogonic charts for the use of the military. In 1946, a chart showing isogonals for epoch 1946 was compiled for the area bounded by latitudes 60°N and 60°S and longitudes 40°E and 168°E. (Anon., 1947).

The two modern theories on magnetism which have been a subject of considerable discussion lately are the "Core Theory" and Blackett's "Bulk Theory". It can be shown that the former theory implies that both H and V (the horizontal and vertical forces of the earth's magnetism) should increase with depth, while according to Blackett's Theory, V should increase with depth but H should decrease. A measurement of the magnetic elements in a deep mine at different levels should thus provide a crucial test between the two theories. Gulatee (1948-49*d*) took observations at six different levels upto a depth of 8700 feet in the Kolar gold fields in 1948. Both H and V were found to increase with depth but the amount of increase in the case of V was much more than that indicated by theory. It was suspected that the anomalous vertical gradient in V was due to the high magnetic susceptibility of the country rocks. To confirm this, a surface magnetic survey covering the entire mining area of the Kolar Gold Fields was undertaken. It was found that the magnetic field in this area was very irregular, and the conclusion was arrived at that the values of H & V in Kolar Gold Field mines are not representative of the radiation with depth of the main field of the earth.

It has been known for some time that the diurnal variations of the horizontal force of the earth's magnetism are enhanced in the areas between the magnetic and geographic equators. At the Oslo session of the International Union of Geodesy and Geophysics in 1948, the International Association of Terrestrial Magnetism and Electricity convened a committee to promote such observations.

The committee put forth the scheme that observations of the range of the daily variation of H should be carried out at a series of stations about 150 km apart lying in a north-south line outside and between the geographic and magnetic equators. The proposal for the observations in India was considered at a meeting of the Central Board of Geophysics and the work was entrusted to the Geodetic Branch of the Survey of India. Four stations were selected in south India and one in Ceylon and special observations were taken in 1950 with three Quartz Horizontal Magnetometers specially lent for the purpose by the International Association of Terrestrial Magnetism. Some important results were obtained and are discussed by Gulatee (1950*b*). It was found that, as expected the ranges of diurnal variation of H are maximum at Kodaikanal and Tinnevelly, which are nearer the magnetic equator than at other stations observed at.

Prediction of Tides

Tide-gauge operations in India were originally commenced in 1873 with a view to settle the controversy which was raging in the press at that time that the Kathiawar coast and the land near the Rann of Kutch were in a state of gradual subsidence. Soon after, the British Association for the Advancement of Science initiated a system of tidal investigations in the United Kingdom and in conformity with it, the Indian Government decided to instal a number of tide-gauges all along the Indian Coast. Starting from the year 1874, automatic tide-gauges (Newman's type) have worked at 42 ports for various periods (normally about five year). Except for a few major ports, the work at all the remaining ports was stopped before the end of the last century.

Tidal observations are required for the following purposes:—

(i) To provide harmonic constants for predictions of tides.
(ii) To provide a datum for the national level net.
(iii) To enable precise values of MSL to be determined and to correlate them with meteorological factors.
(iv) To provide data for deciphering vertical movements of land.
(v) To study the deviations of MSL from a world equipotential surface called the geoid.

The control of tidal observations was entrusted to the Survey of India in 1879, when a tide-predicting machine with 24 components was constructed for the Government of India and set up at the Stores Department, Lambeth, London, and later on at the National Physical Laboratory, Teddington. Data were supplied from Dehra Dun, and the tide tables were published in England till 1921 when the machine was taken over by the Geodetic Branch, Survey of India, Dehra Dun. Since then, the compilation and publication of the annual tide tables of the Indian Ocean have been the concern of the Survey of India. These tide tables contain a total of 67 ports of which 39 ports lying between Suez and Singapore are predicted by the Survey of India and the remaining are obtained on an exchange basis from the Hydrographic Departments of foreign countries. In addition, separate tidal pamphlets are published for Bombay, the Hooghly River, and the Rangoon Rivers.

During the World War II tidal predictions in the form of tidal charts, from which the height of the water can be read at any time, were prepared for quite a number of ports in the Far Eastern waters for operational purposes. These predictions were supplied exclusively to the Eastern Fleet Naval Headquarters and were based on harmonic data of only four main tidal components, M_2, S_2, K_1 and O_1, from which eleven more components were derived theoretically.

Considerable progress has been made in the last few years towards overhauling the older methods of tidal predictions, especially those for riverain ports. The latter require special methods involving considerations of shallow water components to allow for distortion of tidal curves produced by restricted waters. The case

of the Hooghly River ports namely Calcutta, Diamond Harbour and Saugor, requires special mention. From a recent levelling (1949) along the river banks it has appeared that the tidal bench-marks of reference had all undergone considerable changes in their accepted heights due either to subsidence or to local faulty levelling. The gauges have now been adjusted properly and fresh analysis is in progress for these ports.

The dismantling of the majority of the tide-gauge observatories in India was unfortunate. Their important scientific objective (so valuable to the geologists and others) of delineating the relative movements of land and sea appears to have been lost sight of. If a large number of the observatories had been working up to the present moment, valuable material would have accrued for investigating the coastal stability of India. Gulatee (1948) has discussed the necessity for installing more tide-gauges in India for quantitative analysis and research on some important problems.

In pursuance of this, he sponsored the following resolution at the Oslo meeting (1948) of the International Union of Geodesy and Geophysics which was unanimously adopted:

> "The International Union of Geodesy and Geophysics considers that to provide data for a satisfactory study of M.S.L. and its variations on the Indo-Burma-Malaya-Siamese waters and also for detailed studies of many other geophysical problems such as the secular subsidence or elevation of land, the present number of active tide-gauge stations on the Indo-Burma coast is far from adequate, and strongly recommends to the Governments concerned the establishment of a number of additional permanent tide-gauge observatories on their coasts as soon as practicable."

Action is in hand to procure the necessary tide-gauges for the purpose. One gauge has already been installed at Kandla and a proposal is under way to put three or four gauges at different points along the Kandla coast to study the nature of the tides in this regions which pass through a complex network of creeks. In addition, it is hoped to. establish tide gauges at the following ports: Navanar, Keraval, Karwar, Ratnagiri, Beypore, Cochin, Minicoy, Tuticorin, Negapatam, Short Island, Port Blair.

Tide-gauges are already in operation at Karachi, Bombay and Calcutta.

Various scientific bodies, notably the International Association of Physical Oceanography, and other interested in the study of the mean sea level and its fluctuations have been asking the Survey of India for monthly and annual values of the M.S.L. at Indian ports, as obtained from systematic tidal observations for quantitative analysis and research on various problems. These have now been published upto the year 1948. (Anon., 1948-49).

BIBLIOGRAPHY

Airy, G. B. (1855). *Phil. Trans.* **145**, 101.

Anon. (1879). The elements of the figure of the earth. Survey of India, G.T.S., **2**, Chap. X, p. 125.

——— (1910). Levelling of precision in India from 1858 to 1909 Dehra Dun. *G. T. S.*, **19.**

——— (1947). *Tech. Rep. Surv. India*, Part III, Geodetic Work, Ch. III.

——— (1947). *Tech. Rep. Surv. India*, Part III, Ch. VII.

——— (1948-49). *Tech. Rep. Surv. India*, Part III, Ch. V.

——— (1950*a*). *Tech Rep. Surv. India*, Part III, Chart I.

——— (1950*b*). *Tech. Rep. Surv. India*, Part III, Ch. I.

Basevi, J. P. and Heaviside, W. J. (1879). Pendulum operations. *G. T. S.* **5.**

Burrard, S. G. (1901). The attraction of the Himalaya mountains upon the plumb-line in India, considerations of recent data. *Prof. Pap. Surv. India*, No. 5.

Couchman, H. J. (1915). The pendulum operations in India and Burma, 190 -13. *Prof. Pap. Surv. India*, No. 15.

Dessau, G. (1947). Past and future of exploration geophysics in India. *Trans. Min. Geol. Inst., India.*

——— (1948). The geophysical section of the Geological Survey of India. *Indian Min.*, **2.**

Evans, P. and Compton, W. (1946). Geological factors in gravity interpretation illustrated by evidence from India and Burma. *Quart. J. Geol. Soc. Lond.*, **102**, 211-249.

Everest, (1870). Survey of India, G. T., Vol. I.

Glennie, E. A. (1932). Gravity anomalies and the structure of the Earth's crust. *Prof. Pap. Surv. India*, No. 27.

Gulatee, B. L. (1940). Gravity anomalies and the figure of the Earth. *Prof. Pap. Surv. India*, No. 30.

——— (1946*a*). Standards of length. *J. Sci. industr. Res.*, No. 8 (Feb.).

——— (1946*b*). Angular corrections for the Lambert orthomorphic conical projection. *Emp. Surv. Rev.*, p. 311.

——— (1946*c*). Secular variation of magnetic declination in India. *Sci. & Cult.*, Nov., p. 215.

——— (1947*a*). Triangulation in the countries neighbouring India. Part III, Chap. I. *Tech. Rep. Surv. India*,

——— (1947*b*). Geophysical prospecting for manganese near Ramtek. *Mem. Surv. Res. Inst.*, **1**, No. 1.

——— (1947*c*). Geophysical work in India. Conference of Br. Commonwealth Survey Officers, p. 140.

——— (1947*d*). Future of Geophysics in India. *J. Sci. indust. Res.*, Feb.

——— (1948). A note on the desirability of having more permanent tide-gauge observations in India. *J. Sci. indust. Res.*, **7**, 517-520.

——— (1948-49*a*). Subsidence of S. Bengal as evidenced by tidal and levelling operations. *Tech. Rep. Surv. India*, Part III, Chap. VIII, Section II.

——— (1948-49*b*). *Ibid.*, Chap. II.

——— (1948-49*c*). *Tech. Rep. Surv. India*, Part III, p. 146.

——— (1948-49*d*). *Tech. Rep. Surv. India*, Part III, Chap. VI.

——— (1948-49, 1950*a*). *Tech. Rep. Surv. India*, Part III, Chap. III.

——— (1948-49, 1950*b*). *Tech. Rep. Surv. India*, Part III, Chap. III, Para 38.

——— (1949*a*). Levelling in India, past and future. *Tech. Rep. Surv. India*, No. 3.

——— (1949*b*). Topographical—isostatic effect of outer Hayford Zones, *Bull. Geod.*, No. 12, p. 180.

——— (1950*a*). Mount Everest—its name and height. *Tech. Rep. Surv. India*, No. 4.

——— (1950*b*). *Tech. Rep. Surv. India*, Part III, Chap. VI.

Hayford, J. F. (1909). The Figure of the Earth and Isostasy from the Measurements in the U. S.

Helmert (1890). Die Schwerkraft in Hochgebirge.

Hunter, J. de Graaff (1947). Practical application of the Laplace longitude-azimuth relation to control of geodetic anomalies. *Emp. Surv. Rev.*, **9**, 131-134.

Hunter, J. de Graaff and Gulatee, B. L. (1946). The Trans-Persia triangulation 1941-44. War Research Series Pamphlet, No. 9.

Maclear, Thomas (1866). Verification and Extension of La Caille's Arc of Meridian at the Cape of Good Hope. Vols. 1 and 2.

Meinesz, V. (1923-32). Gravity Expeditions at Sea, **2**.

Pratt, (1855). *Phil. Trans.* **145**, 53.

——— (1859). *Phil. Trans.* **149**, 745.

Rao, M. B. Ramachandra (1947). Geophysical investigations for selection of site for Ramapadasagar Dam. *Tech. Publ. Amer. Inst. Min. Engrs.*, Tech Publn., No. 2287.

——— (1949). Recent advances in exploration geophysics. *J. Sci. indust. Res.*

PART III: STATISTICS 1939-1950

By C. RADHAKRISHNA RAO, Ph.D. (Cantab)
Professor of Statistics, Indian Statistical Institute, Calcutta

'Progress is the activity of today and the assurance of tomorrow.'
—Emerson.

CONTENTS

ACKNOWLEDGEMENTS

My thanks are due to Dr. S. K. Banerji, Retired Director General of Observatories, Professor V. M. Dandekar, Gokhale Institute, Poona, Professor K. B. Madhava, Retired Professor of Statistics, Mysore University, Dr. K. R. Nair, Statistician, Forest Research Department, Dehra Dun, Dr. U. S. Nair, Census Superintendent, Trivandrum, Dr. V. G. Panse, Director of Institute of Plant Industry, Indore, Dr. P. B. Patnaik, Head of the Statistics Department, Madras University, Dr. N. S. R. Sastry, Director of Statistics, Reserve Bank, Bombay, Dr. P. V. Sukhatme, Adviser to I.C.A.R., Delhi, Dr. Sundri Vaswani, Statistician, A.T.I.R.A. Ahmedabad, and the staff of the Departments of Statistics, Calcutta and Travancore Universities for supplying the information I asked for.

1. INTRODUCTION

1.1 In this article I begin at the beginning of Statistics in India and briefly survey the developments upto 1938 and proceed to review, in greater detail, its progress from 1939 to 1950. Statistics in the sense of state-craft existed in a highly developed form in ancient India as revealed in the comprehensive compila-

tions of administrative statistics in *Kautilya's Arthasastra* (300 B.C.) and *Ain-i-Akbari* (1590 A.D.). With the growing complexities in the political and socio-economic structure of the country such comprehensive compilations became more and more difficult and the accuracy of the collected information could not be ensured with the result that the official statistics of the recent past have come to exist only as topics for some literary criticism. This was because the older methods of collection were completely unsuited to the changed conditions and statistics as a scientific method of collecting data and extracting information was yet to be born. The first steps in the establishment of statistics on scientific foundations were laid by Francis Galton, Karl Pearson and W. F. R. Weldon towards the end of the last century and by R. A. Fisher in the early twenties of this century. They tried to supply a method, based on the concept of probability, by which one can generalize from the particular, or infer something about a *whole population* from *sample data.*

1.2 For an effective use of these fundamental methods which take into account only the variability of measurements in a population it was necessary to study the nature of errors that arise in the method of measurement itself, the instruments and the human agency involved. An early recognition of this feature of inference is due to Mahalanobis (1923) who studied the "disturbing effect of measurement errors" (see Wold, 1938, p. 206) in the analysis of meteorological data of the upper air. Since then, he developed an interesting method (detailed in section 3) of controlling the errors of measurement and ascertaining their magnitude which is sometimes of a higher order than the variability between the individuals of a population. This appears to be an important development in the collection of statistics and their interpretation, specially under the peculiar conditions existing in India.

1.3 The early work of Mahalanobis in the field of meteorology won for him the recognition of the Government departments who sought his advice on two major schemes for preventing floods in Bengal and Bihar and Orissa. On the basis of statistical investigations it was shown that the proposed schemes of raising the embankments at a cost of 3 crores and 40 lakhs of rupees in the two provinces would not be effective in controlling the floods. Government thus saved money by accepting these recommendations. All this work necessitated the establishment in 1931 of an Institute of Statistics with which was laid the foundation of theoretical research in Statistics in India. Some of the early workers who were responsible for the growth of the Institute which faced initial difficulties, are (late) S. S. Bose, K. B. Madhava and H. Sinha. The following note in *Nature* (1945, Vol. 156, p. 722) summarises the activities of the Indian Statistical Institute, on which depended solely the responsibility of progress of Statistics in India:

"The Institute, as it has now developed, has many facts: on the educational side equally as a training ground for computors and routine statisticians, and as a centre of postgraduate research in the most far-reaching branches of the mathematical theory of statistics and experimental design; as a professional institute and learned society bringing together all schools of thought in Indian Statistics; as an agency employed by departments of Government and advisory bodies, in the essential work of collecting, scrutinizing and digesting the facts upon which administrative decisions must depend. The achievement of co-operation among the many able men needed to guide these various activities has been the work of an applied mathematician, Prof. P. C. Mahalanobis, formerly professor of Physics, acting as honorary secretary to the Institute. He is also a fellow of the Royal Society".

1.4 Immediately after the establishment of the Institute, *Sankhyā*, the Indian Journal of Statistics was published under the editorship of P. C. Mahalanobis, and, to-day, it is regarded as one of the standard journals in Statistics. It is worth quoting from the editorial of the first volume (in 1933) to understand the unique feature of this journal and the various objectives which it maintains even to-day.

"A research journal serves that narrow borderland which separates the known from the unknown, and it is not always possible to see clearly the lines of future developments. We shall, therefore, invite papers of all kinds, appraising them only on the basis of observational accuracy and logical reasoning. We shall publish carefully collected statistical materials irrespective of the subject even if they have not received any analytic treatment. We shall pay special attention to developments of the mathematical theory of statistics, and include abstracts and expositions of important papers published elsewhere. We shall try to help statistical researches on co-operative lines by brining workers in different parts in India in contact, and by providing a medium for exchange of ideas. Bibliographies of Indian Statistical publications, numerical tables tending to reduce the labour of computation, book reviews, and notes and comments on current topics are some of the ways in which we shall try to make Sankhyā useful to statistical workers in India. Knowing that our resources are small we shall seek guidance and help from other countries, and we shall, welcome and thankfully receive papers from abroad.

"The study of modern statistical methods is in its infancy in our country, and we do not expect to be able to achieve immediate results. We shall be satisfied if we can help by our humble efforts to lay the foundations for future work".

1.5 The Indian Statistical Institute was for a long time, the only training centre for advanced students as well as officers on deputation. Mysore University, at the suggestion of its then Vice-Chancellor Brojendranath Seal, instituted a mixed course in Mathematics, Economics and Statistics at the honours level as early as in 1924. Statistics as a full post-graduate course was introduced for the first time in the Calcutta University in 1941. Later Travancore, Mysore, Bombay, Dharwar, Madras, Gauhati, Andhra, Patna and other universities introduced either full or half courses in statistics at the graduate and post-graduate levels. Training courses at the post-graduate and research levels were also instituted in the Indian Council of Agricultural Research at Delhi. The statistical branch of this council is also responsible for project and advisory work on statistical aspects of sampling and experimentation in agriculture, animal husbandry and veterinary science.

1.6 The last decade also saw the growth of statistical bureaux and other coordinating agencies in almost all provinces and states of India. The activities of the statistical bureau in Travancore organised by Dr. U. S. Nair, Professor of Statistics in the University of Travancore, deserve special mention. It is an advisory body which renders advice to government departments, trade associations and other agencies in the collection and interpretation of all types of data. It is also a service agency which carries out agricultural and economic sample surveys and undertakes the statistical analysis of data for recognised private bodies. It is also useful for the university students who are required to take some practical training in the Bureau.

1.7 Some of the research institutions owe their foundation to the philanthrophy of the rich. Gokhale Institute of Politics and Economics is one such institution founded in 1930 on the 25th anniversary of the Servants of India Society and stabilized by the munificient donation of Rao Bahadur R. R. Kale. This Institute conducts research into the various economic and political problems of India and also trains workers for such study and research. The institute has made valuable contributions in methods of collection of data on economic aspects and utilisation of information in framing economic policies and resolving some of the current controversies. They have twenty-two publications uptill now covering a wide range of topics such as demographic studies, farming practices, socio-economic studies and economic policies.

1.8 Two societies called the Calcutta Statistical Association and the Indian Society of Agricultural Statistics have come to be established. They have as their

official organs, the Bulletin of the Calcutta Statistical Association and the Journal of the Indian Society of Agricultural Statistics.

Thus the statistical activities have been gradually expanding to meet the growing needs of the country.

2. Official Statistics

2.1 Great changes have taken place in the collection of official statistics and their presentation since Bowley and Robertson submitted their recommendations in 1935. At that time, to quote their remark, 'though in some branches some work is being done, and determined efforts made to improve the accuracy and scope of information, in others they are unnecessarily diffuse, gravely inexact, incomplete or misleading; while in many important fields general information is almost completely absent'.

2.2 The wide gap in agricultural statistics due to the existence of non-reporting areas is closed partly by instituting permanent agencies for the collection of primary statistics and partly by scientifically planned statistical surevys. The developments in the latter direction are detailed in the next section. Sample surveys are now becoming popular with the Government because of their economy and quickness of availability of the forecasts for Governmental and commercial purposes.

Efforts are being made to secure complete information of the industrial output partly under statutory powers by legislation and partly by a sample census of the manufacturing industries. Similar steps are being taken in the collection of statistics of housing, trading, employment, fuel and power, prices, wages, balance of payments etc. The details of these recent development are given in a paper read by Subramaniam (1947) at the first World Statistical Congress and also in the official publication, *Guide to Indian Official Statistics.*

2.3 The latest advance devise to supply, on all India basis, information on various aspects of rural and urban economy and demographic conditions of the population, is the National Sample Survey being conducted by the Government with the technical assistance of the Indian Statistical Institute. It is hoped that when the survey is complete national income figures with proper break downs into various sectors of the economy will be available for the first time.

2.4 Fortunately for India proper co-ordination is growing up between 'official and academic statistics or to speak functionally, between the duties of collections, enumeration, tabulation and publication, which absorb the time of official statisticians and the duty of study, analysis and interpretation which falls to the duty of mathematical or theoretical statistician'. An inter departmental committee of statisticians and economists set up by the Government of India critically review, from time to time, the position of official statistics, the methods of collection and their utility, and make suitable recommendations.

A Central Statistical Organisation is also being set up to co-ordinate the statistical activities of various Governmental departments and help them in the collection of information. With such a set up in addition to a Statistical Advisor to the Cabinet, the statistical services in India are now on solid foundations.

3. Sample Surveys

3.1 Research in sample surveys has long ceased to be derivation of formulæ for best estimates and standard errors. Notwithstanding the simplicity in its theory, in practice it offers an intractable problem. There is much to be assumed before the theoretical statistician could regard his data as having arisen from a mathematical model. A stock taking will reveal the wealth of information pain-

stakingly acquired during the last decade and how it is being utilized in the solution of this complex problem.

3.2 *Early crop cutting experiments.* An account of early crop-cutting work in India carried out in 1923, 1924, and 1925 is contained in an important paper by Hubback (1927) reprinted in Sankhya, 7, 281, with a critical note by Mahalanobis (1946a). This publication is remarkable in several ways.

(i) It emphasized the necessity of random sampling for determining the extent of error in the estimate.

(ii) It drew attention (a) to the differences that may exist between area sown and area harvested and (b) to the correction needed for the area occupied by ails and recommended that harvested area could, with advantage, be obtained from some sort of sampling.

(iii) It pointed out that sampling for yield could not be simple in practice and should necessarily be in the nature of what is now called two stage.

(iv) It recommended the use of a sample cut of size 1/3200 of an acre for yield estimation from the point of view of practicability and provided a theoretical justification for it by introducing the variance function.

The theoretical investigations supplemented by emperical verification from Hubback's own data provide a fascinating study as a piece of original thinking. R. A. Fisher mentioned that Hubback's work greatly influenced the development of his work at Rothamstead.

Sometime later the method developed by Hubback was used in the Central Province by C. D. Deshmukh and P. S. Rau, a member of the Indian Civil Service.

3.3 *Acreage surveys.* Then came the series of large scale sample surveys for acreage developed in the Indian Statistical Institute. The design adopted was stratified unistage sampling, the ultimate unit being a cluster of 'fields' chosen with probability proportional to the area of the cluster. The cluster area was sometimes made uniform by considering portions of fields falling in a square frame of optimum size 2.5 acres. The essential features of these developments were detailed in a paper by Mahalanobis (1944).

(i) For the first time the importance of pilot surveys and in general the sequential method of successively improving the design by using the information supplied by the previous surveys have been stressed. The money spent on the complete enumeration of two thanas, to begin with, yielded a good dividend because the pilot survey provided the basic material viz., the estimates of cost and variance functions in terms of the shape and size of the sampling unit and the number of units to be chosen, and with these an optimum design was derived minimizing the expenditure for a given margin of error (Mahalanobis, 1939).

(ii) It must be remembered that area under a crop refers to crop acreage at a particular point of time. In fact there is considerable difference between area sown and area harvested and it is important to trace the changes in the area under a crop through a season. This introduced a new concept of randomization over space and time by which the history of the crop from the stage of early sowing to late harvesting could be studied (Mahalanobis, 1945).

(iii) A third feature of these surveys was the use of a highly trained staff of mobile investigators who travel from place to place and collect information on the sampling units. It may be noted that a highly trained staff once recruited will be useful not only for acreage surveys but for multipurpose surveys (Mahalanobis 1946*b*) which cannot obviously be carried out with the help of any Government agencies now existing.

(iv) A fourth feature of these surveys was the use of interpenetrating network of samples by which independent estimates could be obtained and the

validity of sample surveys checked (Mahalanobis, 1944). This principle has been adopted as an essential requirement of any sample survey by the expert committee on statistics of the United Nations (see the report of the Committee reprinted in *Sankhyā,* **9,** 392).

The importance of the use of interpenetrating net-work of samples has, however, been questioned on various grounds by Sukhatme and Panse (1948), Ghosh (1949*d*) and others.

(i) It involved greater expenditure without increasing the precision of the estimate.

(ii) Sufficient data could not be obtained to secure efficient comparisons within each stratum except at a very prohibitive cost.

(iii) One was at a loss to know what to do when two estimates differed.

(iv) Even if two estimates showed agreement there was not enough justification to feel that they were nearer to the true value.

It may be observed that a satistician who has two independent estimates is unquestionably in a better position than the one having only a single estimate so that the only valid criticism could be that interpenetrating sub-samples involved a higher expenditure.

An alternative to this has been suggested in that adequate supervision should be provided to ensure reliability. Even in this situation the statistician faces a similar problem. The investigator supplies a series of figures and the supervisor another series corresponding to some of the sample units covered by the investigator. These two interpenetrating samples are also supplied by two human agencies. It may be noted that the simplest type of interpenetrating samples as devised by Mahalanobis (1944) consisted of samples in groups of n_1, n_2 and n_{12} units with one party. A investigating the first and the third groups and another party B, the second and the third groups so that duplicate figures were obtained for some of the sample units. To the academic statistician who cannot distinguish between the supervisor and the investigator on any ethical grounds the argument of adequate supervision seems inconvincing. The practical statistician lost in the controversy is hoping for further investigation into the problem of providing adequate checks in large scale sample surveys.

3.4 *Large scale surveys for yield.* The Indian Statistical Institute has been working since 1938 on the problem of improving the estimate of crop yields. During the season 1938-39 it helped the Government in an extensive crop cutting experiment on paddy covering about 1000 square miles of Burdwan-Hooghly-Howrah Flushing and Irrigation Area (Mahalanobis, 1937). The experiment was repeated in 1939-40 and 1940-41. Crop-cutting experiments on sugarcane and wheat were carried out in U.P. in 1940-41 and 1941-42. Later, in Bengal, crop-cutting experiments on jute and paddy were undertaken by the Institute. During the last 10 years various experiments have been specially designed to determine the optimum shape and size of the sample cuts. It was found (Mahalanobis, 1944) that cuts of size over 100 square feet were adequate and convenient in practice. As for shape a circular cut devised by J. M. Sengupta was found to be best suited for crop-cutting work.

In 1942-43, and 1943-44, Panse and Kalamkar estimated the yield of cotton in Central Provinces and Berar by random sampling method using the departmental staff. At the instance of the Government of India, the I.C.A.R. (Indian Council of Agricultural Research) undertook to conduct yield surveys of food crops from 1943 under the leadership of P. V. Sukhatme. The design adopted was stratified two stage random sampling, the districts forming the strata, the villages within a district, the first stage sampling units and the fields within a village, the second stage sampling units (Sukhatme, 1945). As in the surveys

conducted by Panse and Kalamkar (1945) the crop-cutting work was entrusted to the staff of the agricultural department. The simplicity of the design led the State Government to accept readily the above scheme and at present, in most of the Indian States the yield surveys of crops like wheat, paddy, gram, *masoor, jawar, tur etc.*, are being conducted with the technical assistance of the I.C.A.R. This should be considered a notable achievement of Sukhatme and his associates at the I.C.A.R. for although various committees and commissions has been recommending sampling procedures for crop yield for the last 40 years or so the Government machinery could not be easily moved to accept them.

There is some difference of opinion about the size of the sample cut, Sukhatme (1946, *a,b,c,* 1947 *a,b*) contending that the cut size should be as large as that (500 to 900 sq.ft.) used in the I.C.A.R. surveys to obtain unbiased estimates and Mahalanobis (1946*c*) pointing out that it is wasteful to go in for cuts for such a large size. The evidence as it stands is not conclusive enough to decide one way or the other. The problem is not simple either. Questions of bias, variance, validity and above all practical convenience have to be considered with special reference to the agency employed. It is time that some further investigations are planned in this direction.

3.5 *Socio-economic surveys.* From 1935, a number of socio-economic surveys have been organised by the Indian Statistical Institute, the most important of which are a survey of the economic conditions of hand-loom weavers in Bengal (Chakravarti, 1937), Jagaddal Labour Enquiry (Mahalanobis, 1942) where effective controls were employed to determine investigator bias, an enquiry into the prevalence of drinking tea (Mahalanobis, 1943) and a Radio and Public preference survey (Mahalanobis, 1941*a*). A sample survey of famine conditions conducted by Chattopadyaya, Mahalanobis, Mukherjee and Ghosh (1936) revealed the extent of damage done to rural economy by the famine conditions in Bengal in 1943. An important feature of this report is the demonstration by the authors that the famine of 1943 was not a chance phenomenon but the culmination of the progressive deterioration of Bengal's rural economy over a long period. The Jagaddal Labour Enquiry is being continued from year to year with some families remaining common in all the surveys. By this method efficient estimates of changes in economic conditions can be secured. Some results obtained in this connexion will be shortly published.

By far the most important and extensive multipurpose survey now in progress is the National Sample Survey where 1800 villages spread all over India will be visited and information on socio-economic and demographic conditions of a number of households will be obtained.

3.6 *Some special techniques of sample surveys.* The estimate of bark yield of Cinchona plants in a block of plantations presented a new problem because of the difficulty in barkng a sample plant and obtaining its dry weight. It was demonstrated that an efficient sampling plan would be, first to obtain by a study of a few plants the prediction formula for the weight of dry bark in terms of the external (or concomitant) measurements on a plant such as its height, girth, number of branches and the weight of cork borings. Then mean values of these characteristics are estimated from a large sample of plants and substituted in the prediction formula to obtain the average yield of bark per plant. The problem is not solved until the number of plants in a block could be ascertained. Although the number of plants sown is known the survival rate is not known. This needed line and block samplings of a systematic type. The details are given by Mahalanobis (1941*b*). The final estimate of yield is of the form

$$P\ (a+b_1\bar{x}_1+b_2\bar{x}_2=\ldots)$$

where p is the estimated number of plants, $\bar{x}_1$, $\bar{x}_2$,...are sample estimates of the

means of concomitant measurements and b_1, b_2,...are the regression coefficients. The variance formulae have been given by Chameli Bose (1943), and Ghosh (1947*a*).

The method of multistage sampling has been successfully employed by Banerji (1948) in the estimation of white fly incidence in sugarcane. Various devices used in forest sampling have been critically discussed by Nair (1950).

3.7 *Theoretical research in sample surveys.* Topographic variation which deals with the nature or pattern of special distribution of some variate or variates was the main topic extensively investigated by a number of workers. This is important in determining the optimium type of sampling in both large scale aerial surveys and also controlled field-polt experiments. The theoretical investigations have, however, a wider field, being useful in the study of time series, fluid-turbulence and other problems.

The first attempt in this direction was made by Mahalanobis (1944) in India. By model sampling experiments he found that the variance associated with a sample unit consisting of n elementary units in some pattern is proportional to n^{-g} where $0<g<1$, the value 1 being attained when the space correlation is zero and 0 when the space correlation is perfect. This formula was used in deriving the optimum combination of the elementary units to form a sampling unit.

Ghosh (1949*b*, 1949*c*) developed special techniques in the general study of topographic variation and tests of randomness. He has constructed some stochastic models for some natural fields, e.g., multistage non-stationary models (1943*e*) and stationary models with exponential topographic correlations (1949*a*). These models satisfy the requirements that the topographic correlation in any direction is a 'non-negative, monotonic, non-increasing function' of the distance with the correlogram concave upwards. He also studied the topographic correlations for some artificial fields (1943*b*), the alternative shapes for basic cells in two and higher dimensional fields (1943*d*, 1947*b*), and the distribution of random distances within and between rectangles (1943*a*, 1943*c*). Ghosh (1950) extended his work on topographic variation to a type of weighted variate.

Das (1950) demonstrated that in multi-dimensional sampling, if the space correlation is a decreasing function of distance and the correlogram is concave upwards then systematic, stratified and random sampling are in decreasing order of efficiency. Sukhatme (1950) derived a general formula to estimate the gain in precision due to stratification in a sub-sampling design for finite populations. A general approach has been indicated for calculating the relative efficiency of sampling units of different size of one versus two stage sampling.

4. Experimental Designs

4.1 *Early work on field experimentation.* The Indian Statistical Institute took the initiative in India to introduce the techniques of field experimentation originally developed by R.A. Fisher in the twenties of this century. From 1931 to 1937 various experimental designs such as Latin square, factorial arrangements with an without confounding split-pot and quasi-factorial arrangements have been tried. Uniformity trial data were extensively analyzed to determine the optimum shape and size of the plots for experimentation. All this earlier work both on the theoretical and practical aspects was due to S. S. Bose and P. C. Mahalanobis who had been exploring a new field of fundamental importance to an agricultural country like ours. Two of their joint papers published in 1933 and 1935 deserve special mention.

4.2 *Designs for varietal trials.* By this time F. Yates introduced a new type of varietal design called the balanced incomplete block which is equivalent to a configuration of v elements in b sets of k each, such that each element is used r

times and any pair of elements occurs in λ sets. Bose (1939) developed systematic methods of constructing these designs and provided some new solutions. He found that when $v = (p^n - 1)/(p-1)$ or p^n, p being a prime, the balanced incomplete block design is equivalent to Projective or Euclidean finite geometrical configurations. Since such geometries exist for prime p a huge class of designs was made available.

A second method employed by Bose was the method of differences by which starting from an initial set or sets of elements the whole arrangement could be derived by successive cyclic addition of the elements. Bose (1942) proved an affine analogue of Singer's theorem and derived a number of cyclic solutions to the designs following the Euclidean plane geometry. Later Rao (1945, 1946*a*) extended the results of Singer and Bose to finite geometrical configurations of higher dimensions and provided cyclic solutions to a number of designs. Most of these representations are now incorporated in the table of designs in Fisher and Yates statistical tables. K. N. Bhattacharya (1944, 1946), by an ingenious device, obtained new solutions for the designs with $\nu = b = 25$, $r = 9 = k$, $\lambda = 3$; $\nu = 16$, $b = 24$, $r = 9$, $k = 6$, $\lambda = 3$; $\nu = 31 = b$, $r = 10 = k$, $\lambda = 3$ and $\nu = 21$, $b = 30$, $r = 10$, $k = 7$, $\lambda = 3$. K. N. Bhattacharya (1950) and Nair (1950) constructed several designs of the partially balanced incomplete block type described below. Shrikande (1950) proved the impossibility of certain symmetrical balanced incomplete blocks. Following the method of differential operators developed by Mac-Mohan, Saxena (1950) enumerated the different types of Latin Square arrangements of order six and confirmed the figure arrived at by Fisher and Yates earlier.

4.3 The work of the Indian authors was not confined to the construction of balanced incomplete block designs alone. This type of designs provides only a limited number of experimental arrangements because of the severe combinatorial restrictions. Arrangements are needed to suit different numbers of varieties. A significant advance in this direction is the introduction of partially balanced designs by Bose and Nair (1939) and later generalized to cover a wider variety of designs by Nair and Rao (1942*a*).

The arrangement is as follows:

(a) There are v varieties to be tested in b blocks of k plots each variety being replicated r times.

(b) Given any variety the rest fall into groups of sizes $n_1, n_2, \ldots, n_m$ called the 1st, 2nd,... and mth associates such that the given variety occurs with any one in the i th group in λ_i blocks. It is not necessary that all λ_i should be different.

(c) If p^k_{rs} is the numer of varieties common to the rth and sth associates of a pair of varieties which are k th associates then this should be the same for all pairs of k th associates.

The above design does not completely cover all the situations. For instance a design may be needed where two sets of varieties are used and all comparisons within sets should be of equal efficiency though different from set to set. The design should also provide efficient comparisons of varieties between sets. For this purpose Nair and Rao (1942*b*) introduced a new design called the intra and inter-group balanced design. The requirements are that every variety in the i th group is replicated r_i times and every pair of varieties in the ith group occurs in λ_{ii} blocks and every pair consisting of one variety each from the i th and j th groups occurs in λ_{ij} blocks. Work on the construction of designs is in progress.

4.4 Systematic methods for the analysis of these designs with respect to estimation of missing or mixed up plots, intra and inter-block information have been

developed by Nair (1939, 1944*a, b*) and Rao (1947*a*). Nandi (1947) also discussed a mathematical model leading to inter-block estimation.

4.5 A new class of designs with blocks of unequal size has also been developed by Kishen (1941). Although there is some theoretical difficulty in taking into account the unequal variances arising out of unequal block sizes the above designs are useful for experimentation in hilly tracts etc., where blocks of uniform size cannot be laid.

4.6 *Designs of factorial experiments.* The work on factorial designs was started by Nair (1938) who extending the work of F. Yates provided confounded factorial arrangements when the levels of factors are 4 and 5. Later a systematic study was undertaken by Bose and Kishen (1940) who developed a general theory for deriving all possible confounded designs. The s^m treatment combinations are identified with the points of the Finite Euclidean geometry E G (m, p^n). The contrasts between the sets of treatments corresponding to the points of the s different $(m-1)$-flats of a parallel pencil carry $(s-1)$ degrees of freedom and as the number of such parallel pencils is $(s^m-1)/(s-1)$ all the degrees of freedom are accounted for. This led to a method of completely enumerating the various possibilities of confounded designs in blocks of s^k plots $(k<m)$. Bose (1947) has shown, by following the same representation as above, the possibility of constructing the minimum number of partially confounded arrangements to achieve balance on the loss of information on every contrast belonging to a specified order interaction. Rao (1946*b*, 1947*b*, 1950) defined a new type of arrangements called 'hypercubes of strength d' and showed that these arrangements provide (i) designs for symmetrical factorial experiments involving the maximum number of factors with only interactions of order greater than d confounded, (ii) multifactorial designs when higher order interactions are assumed to be non-existing and (iii) fractionally replicated factorial designs.

4.7 Kishen (1942, 1950) developed a general method for expressing any single contrast for treatments in the case of the general symmetrical factorial design s^m in terms of contrasts for main effects and interactions derived from finite geometrical configurations and utilized this for obtaining unitary components for interaction of any given order. Kishen (1949) also used higher dimensional finite geometries in constructing Latin and Hyper-Graeco-Latin cubes which are natural extensions of the Graeco-Latin square of Euler. These may be useful in certain types of experiments. Kishen (1947) gave a general theory of practical replications in the case of s^k design and derived a number of useful arrangements.

4.8 Ramamurthi and Sitaraman (1942) derived a formula for determining the maximum order interaction below which all can be preserved in a 2^n fractional confounded design in blocks of 2^k plots. Bhattacharya (1942) extended this result to the general s^n design in blocks of s^k.

4.9 Rao (1946*c*) discussed methods of constructing confounded factorial designs in Quasi-Latin squares by which soil heterogeneity could be eliminated in two directions.

Nair and Rao (1948) gave a general method of deriving confounded designs in the case of asymmetrical factorial experiments. The combinatorial problem involved in this investigation satisfies the requirements of a partially balanced varietal trial as generalized by these authors. This was a first step towards the unification of all experimental arrangements factorial and varietal under one combinatorial structure. The two generic designs giving rise to various types of experimental arrangements are diagrammatically represented below.

EXPERIMENTAL ARRANGEMENTS

- Generalized partially balanced, Nair & Rao (1942*a*)
  - Confounded factorial experiments
    - Symmetrical
    - Asymmetrical
  - Varietal trials
    - Quasi-factorial
    - Lattice
    - Other types
- Intra-Inter-group balanced, Nair & Rao (1942*b*)
  - Other types
    - Symmetrically balanced
  - Method of controls

4.10 *Weighing Designs.* It has long been recognized that by weighing a number of objects in suitable combinations higher accuracy can be achieved in the determination of weight of each individual object. If there are p objects with hypothetical weights β_1, ..., β_p then the results of weighing a given combination

$$x_1\beta_1+x_2\beta_2+\ \dots\ +x_p\beta_p=y+\epsilon$$

where y is the observed weight ϵ is an undetermined error and $x_i=+1,-1$ or 0 according as the i-th object is placed in the left pan, right pan or in neither. When a spring balance is used x_i can take only two values 0 and 1. The formulæ for estimates and errors are a direct consequence of the theory of linear estimation discussed elsewhere (section 7). But the problem of designing that is the determination of the number and nature of efficient combinations to be weighed or the determination of the matrix X of the observational equations remained to be tackled. Following the work of Hotelling, Kishen (1945) provided a general method of deriving optimum designs when p is of the form 2^s Banerji in a series of articles (1948, 1949*a*, 1949*b*), made a systematic study of this problem exploring the possibilities of deriving weighing designs from the arrangements of incomplete blocks and factorial experimental designs. He also discussed the possibility of arriving at uncorrelated estimates of the weights and showed that by using some particular types of designs the variance of each estimated weight can be made as small as σ^2/n with a total of n weighings whereas to achieve this much accuracy by weighing the objects individually n weighings will be needed for each object or a total of np weighing. In these cases the problem of designing is solved. But the problem of optimum designing in the general case is not completely solved.

4.11 *Silvicultural experiments.* A brief historical sketch of the introduction of statistical methods in Indian silvicultural experiments is given by Nair (1950). The publication of the experimental manual in 1931 by Mr. Champion marked the starting point in standardizing methods of experimental research in Indian forestry. The silviculturists were introduced to Fisherian techniques in design of experiments from which valid conclusions could be drawn. In 1947 Griffith and Sant Ram published a more comprehensive statistical manual as Vol. 2 of the revised 'Silviculture Research Code'. This is in three parts. Part I gives general principles and includes only as much mathematics as is found absolutely essential. Part II describes the design of forest experiments and Part III consists of a number of typical examples of the working out of typical forest experimental data.

To meet the expanding needs of the Forest Research Institute at Dehra Dun a provision was made for the creation of a statistical branch headed by an expert

statistician. This branch was brought into existence in 1947 and is doing invaluable service in planning of experiments and analysis of data according to modern statistical principles.

5. Multivariate Analysis

5.1 *Distance functions.* Interest in the utilization of multiple measurements arose in India with special reference to problems of classification of different groups for purposes of discovering racial affinities. As early as in 1911, Dr. Sir Brajendranath Seal, in his address on *Race Origin* delivered before the first Universal Races in London, proposed a geometrical representation. He had stated, "If the groups requiring to be arranged vary in n characters and if biometric measurements are complete, the composite mean of the groups may be taken as the point of origin, and then the means of the single characters for each group may be imagined as marked off on n coordinates, and the position in n dimensional space of each group could be easily assigned". Following this notion Mahalanobis (1930) devised a generalized measure of distance D^2 between two groups which corresponds to the square of the Euclidean distance in an n-dimensional space with oblique axes, the angles being determined by the corre'ations of the characters. The distance so defined seemed to have a distinct advantage over the coefficient of racial likeness (C.R.L) proposed by Karl Pearson for purposes of classification because C.R.L. does not take correlations into account. Recently Rao (1948*b*) defined a general distance function as a decreasing function of the amount of overlap between two populations, or the frequency of individuals who are liable to be misclassified by following the best discrimination procedure. Higher distances are associated with smaller frequencies of error in classification. This measure which appears to be well suited for purposes of classifying the groups themselves on the basis of their affinities reduces to Mahalanobis D^2 in the case of normal populations, thus affording further justification to the use of D^2.

5.2 Bhattacharya (1946) defined the distance between two discrete population in k classes characterized by two sets of proportions $\pi_1, \ldots, \pi_k$ and $\pi_1', \ldots, \pi'_k$ as $\cos^{-1} (\Sigma \sqrt{\pi_i \pi_i'})$ and in the case of continuous populations with densities $\phi\ (x)$ and $\psi\ (x)$ by $\cos^{-1} \int \sqrt{\phi\psi}\, dx$. When the distribution is parametric, Rao (1945) considered the metric $\Sigma\Sigma\, g_{ij} d\theta_i d\theta_j$ defining the element of length in the parametric space, where (g_{ij}) is the information matrix. The distance between any two populations is the length of the geodesic distance between the parametric points defining them. All these definitions reduce to Mahalanobis D^2 in the case of multivariate normal populations.

5.3 *Problems of multivariate distributions.* For an ecective use of D^2, its distribution and its properties as an estimate of the corresponding population parameter must be known. If $\bar{x}_i$, $\bar{x}'_i$ represent the mean values of i-th character in the two samples and (a^{ij}) the reciprocal of the estimated dispersion matrix, then

$$D^2 = \Sigma\Sigma\ a^{ij} (\bar{x}_i - \bar{x}'_i) (\bar{x}_j - \bar{x}'_j)$$

The distribution problem was first solved by Bose (1936) in the classical form and by Bose and Roy (1938) in the Studentized form.

In solving these distribution problems Mahalanobis, Bose and Roy (1937) found some general methods in obtaining multivariate distributions with the help of what are called rectangular coordinates. Wishart's and allied distributions were shown to follow directly from the distribution of the rectangular coordinates. Narain (1948) has demonstrated the further use of the rectangular coordinates in deriving the distributions of the multiple correlation, D^2 etc.

5.4 *Introduction of new test criteria.* In the application of D^2 one assumption is made that the groups under consideration possibly differ in the mean values but not in the variances and covariances. To test this hypothesis Roy (1939)

introduced some test criteria depending on the roots of the determinantal equation

$$|s_{ij} - \lambda s'_{ij}| = 0$$

where (s_{ij}) and (s'_{ij}) are estimated disperson matrices. In a series of papers Roy (1942, 1945, 1946) obtained the distributions of individual roots and also of their symmetric functions in the null and non-null hypotheses for the general case of p variates. By this time R. A. Fisher introduced the roots of determinantal equation for testing differences in mean values when the dispersion matrices are considered to be the same. The distribution problems are the same on the null hypothesis so that Roy's work has become useful in both the types of problems. The problem of selecting the most useful function of the roots for purposes of testing is under consideration and Roy has already some interesting results to publish. Nanda (1948*a*, 1948*b*, 1949) working on similar lines derived the distributions of the individual roots and also of their sum.

5.5 A further problem which arises in the use of D^2 and allied applications is the choice of characters, their number and nature. The question to be asked is whether some assigned characters show any discrimination between the groups independently of a basic pannel of characters already selected. It is to be recognized that these assigned characters may be important by themselves for purposes of discrimination but, in conjunction with a basic pannel, may not contribute any additional distance. This is important if a stable classification of a given number of groups is to be obtained. There must be a limit beyond which the rest of the characters do not contribute to the distance function. This has been stated in an axiomatic manner by Mahalanobis (1937). An emperical test is now available. The test criterion generalized T, for testing simultaneously the differences in p characters was first given by H. Hotelling and is connected with Mahalanobis D^2 by a simple relationship. For the problem of additional discrimination Rao (1946) proposed the test criterion $1 + U_{q,p} = \dfrac{1+T_{p+q}}{1+T_p}$ which is some sort of a ratio of the estimated distances for p and $p+q$ characters. The value of U is small when the additional characters are of no use. It was shown that $U_{q,p}$ is distributed independently of T_p so that the test is independent of the differences in the first p characters. Later he (1949) introduced another statistic $W_{q,p} = T_{q+p} - T_p$ which is useful mainly in situations where T_p is not significant by itself. The relative merits of U and W are considered in some detail in this paper. The non-null distributions of U and W were derived by Rao (1949, 1950). Narian (1949) also obtained the non-null distribution of U. A number of practical applications and the treatment of similar problems for more than two groups by analysis of dispersion and Wilks Λ criterion have been given by Rao (1948*a*). An asymptotic expansion for the exact distribution of Λ has been given in a suitable form with Bartlett's approximation for the first dominating term and correction factors of rapidly decreasing order for the other terms. He also obtained an alternative expansion in a beta series with the first term providing a powerful approximation the second term being $O\left(\dfrac{1}{n^4}\right)$.

5.6 *Problems of discrimination.* One of the problems confronted in biometric research is the assignment of an individual to its proper group. A jaw bone may be found and it is required to find the sex of its possessor, or a newly discovered fossil has to be placed in the proper group. In the case of two alternative groups R. A. Fisher suggested the use of the discriminant function which is a linear combination of the measurements. Rao (1948*b*) discussed the general theory when the alternatives are more than two. Two distinct problems arise depending on the existence or non-existence of the apriori probabilities. In the former case the solutions are derivable from Baye's theorem. In the latter case

optimum solutions have been obtained when the ratio of errors allowable for various groups is assigned. Where it is necessary to assert confidently that an individual has been correctly classified it was shown that the region in the character space favourable to individuals of any group can be further divided into two subregions such that an individual falling into one is confidently asserted to belong to that group while in the second he is only provisionally assigned. These subregions are independent of the apriori probabilities of the various groups.

5.7 *Computational problems.* The multivariate computations are extremely complicated because of the inversion of higher order matrices, solutions of equations and the evaluation of quadratic forms. The complications arise mainly in taking into account the correlations between the characters. Attempts have been made to effect a transformation of the characters to an uncorrelated set. Two simple methods with automatic checks useful in two different situations have been illustrated by Rao in his 1949 and 1950 papers.

5.8 *Applications.* Extensive use has been made of D^2 (see section 9 for details) in the analysis of the United Provinces anthropometric measurements. A significant classification has emerged out of these studies.

An interesting application of multivariate analysis in the study of chronology of Shakespeare's plays was given by Yardi (1946). Considering four aspects of style which appeared to have some chronological significance, Yardi found the best discriminating function between 4 groups of plays which referred to 4 distinct periods. This alone was not sufficient to give the succession of plays within each period. By knowing the dates of some plays it was possible to set up the regression function of time as the best discriminating score and then interpolate for time at the score of a play whose date was not known. Another application due to Rao and Slater (1949) is in the classification of neurotic diseases. It was shown that the variation between different neurotic groups is predominantly confined to one dimension showing that differences exist only in the degree of neurosis with the normal group at one end and obsessionals at the other, personality change, anxiety state, hysteria and psychopathy coming in between. A light variation found in the second dimension was confined to the difference between obsessionals and psychopaths and this admitted a psychological interpretation of the contrasting scores for inadequacy and instability of these two groups.

6. Theory of Distributions and Problems of Tabulation

6.1 *Some problems of distribution.* A notable advance in the theory of distributions is the application of moment functions in the study of distribution laws by U. S. Nair (1939). He used Mellin's inversion transform to derive the probability density of any statistic from its moment function. This is, sometimes, simpler than the characteristic function approach because the expressions for the moments are easily derivable. Using this method Nair derived the exact distribution of the L statistic originally proposed by Neyman and Pearson for testing homogeneity of a number of estimated variances. George (1945) adopted the above method in deriving the exact distribution of L in some particular cases and studied the accuracy of different approximations.

6.2 The sampling distribution of a number of statistics have been obtained by Sukhatme (1936*a*, 1936*b*, 1837*a*, 1837*b*, 1938*a*) for tests of significance connected with exponential and chi square populations. General expressions for the moments and product moments of moment statistics for samples drawn from finite and infinite populations have been obtained by Sukhatme (1943). Earlier he (1938*b*) gave a proof of Fisher's combinatorial method of obtaining the moments and cumulants of the distribution of *k*-statistics. The bipartional functions used in this connexion were derived in another paper by Sukhatme (1939).

Chakravarty (1946, 1947) derived the moments and semi-invariants of the mean square successive difference and derived its exact distribution for $n = 3$.

From parallel samples of sizes $n_1, n_2, \ldots$ providing estimates of means $\bar{x}_1, \bar{x}_2 \ldots$ and variances $s_1^2, s_2^2, \ldots$ Krishnasastry (1946) derived the distribution of the statistic $\Sigma a_i x_1 / \sqrt{\Sigma n_1 s_1^2}$ where a's are unrestricted constants.

Krishna Iyer (1945) gave alternative derivations of the distribution of some well known statistics by the use of a generalized Dirichlet's integral. Bhargava (1946) developed a test for intra-class correlation when the family sizes are different.

Ghurye (1949) compared the relative merits of the different transformations of the binomial variate and studied their validity in small and large samples. Sundri Vaswani (1950) provided a test to verify whether a contigency table could have arisen from a trivariate normal distribution. This was done by fitting 6 constants corresponding to three planes of section and three correlations in which case the x^2 goodness of fit has $7-6=1$ degree of freedom. Uttamchand (1950) considered the problems of testing the quality of two means and two regression coefficients when the residual variances are unequal and investigated the question of utilizing an approximately determinate knowledge about the unknown ratio of variances.

6.3 *Tests of randomness.* M. N. Ghosh (1947, 1948) has demonstrated that, analogous to the serial correlation, the space correlation as defined by Bojarski and Mahalanobis has asymptotically a normal distribution among the set of permutations of the observations. Further generalization of this work leading to the construction of efficient non-parametric tests of randomness is in the press.

For testing the randomness of a space distribution Mahalanobis proposed the use of patch number, a patch being defined as the combinations with side contact of a number of square cells (the whole space being divided into such cells) possessing the same characteristics. Considering only two characteristics, say, black and white and assuming that the probability for any cell to be black is p and to be white q, $(p+q=1)$, Bose (1946) derived the mean and variance of a quantity X defined to be the number of white patches minus the number of embedded black patches.

Krishna Iyer (1948*a*, 1948*b*, 1949*b*) developed the general theory of random association of points in the p dimensional latice $l_1 \times l_2 \times \ldots \times l_p$. Each point can assume one of k colours. Sampling may be free, in which case each point can have independently the k colours with fixed probabilities $p_1 \ldots\ldots p_k$ or non-free, in which case the totals for colours are fixed and all arrangements consistent with this requirement are equally likely. The statistics used are the number of joins of (a) the same colour, (b) two specified colours and (c) two different colours. A join may be confined to adjacent points in a particular direction, say, along an axis of the lattice or to all adjacent points. These two cases are useful for practical applications. Finally the number of doublets triplets and s-plets are also considered. An s-plet is a combination of s points all joined together and an s-plet may have any configuration. In the case of a line it refers to a length or run of joins. In these cases Krishna Iyer derived moments and studied the approach to normality. His theorem (1949*a*) on the evaluation of moments is of particular interest. Some of these results are applied for testing the departure from randomness of a given distribution of diseased plants in a field. Krishna Iyer (1950) also studied the limiting forms of the distributions of joins and runs of a given length when each point can possess one of k colours where $k>2$. B. V. Sukhatme (1949) considered the sampling distribution of the number of triplets of diseased plants in a block of plantations arranged in rows and columns and suggested its use in testing whether the spread of the disease is random or not.

6.4 *Effects of non-normality on standard tests.* Pearson and Adyanthayya (1929), Krishnan Nair (1941), Cheriyan (1945) and others approached the problem of studying the effects of non-normality on standard tests by model sampling experiments. The investigations were intended to show that t and F tests become unreliable only when departures from normality were considerable. Gayen (1949, 1950*a*, *b*,) considered probability densities expressible approximately in Edgeworth series up to terms containing third and fourth moments or β_1 and β_2 and derived t, F and r distributions. The appropriate corrections in terms of apriori values of β_1 and β_2 have been extensively tabulated. These results will enable one to examine the probabilities for *a priori* values of the measures of skewness and deviations from normal kurtosis. When the exact values are not available one may safeguard against error by considering the corrections for their estimated values. The distributions derived by Gayen enjoy an asymptotic property in being (i) valid for large samples from any universe and (ii) sufficiently accurate for any size of samples from a moderately non-normal population. Satisfactory agreement between the theoretical values and experimental results when they are adequately obtained appears to confirm this conjecture.

6.5 *Limiting distributions.* M. N. Ghosh (1946) has given an expression for the order of approximation involved in the application of Laplace Liapounoff's central limit theorem for the case where the absolute moments of order $2+|\delta|$ are assumed to exist. If $\phi(x)$ and $U_n(x)$ denote the normal approximation and the actual distribution function of $x_1+\ldots+x_n$ then

$$|\phi(x)-U_n(x)| < \left(\frac{3}{\sqrt{2\pi}}+1\right) n^{-(p-2)/(11p-14)} + \sqrt{2\pi e}\,\mu_p\, n^{-(5p-2)/(11p-14)}$$

which gives useful results for $2<p<3$. His further work on the generalization of the second limit theorem of Frechet and Shohat regarding the convergence of the set of moments to the case where the distribution functions depend on a random element and ordinary convergence is replaced by convergence in probability, is in press.

6.6 A general theory of large sample tests using the likelihood functions was developed by Rao (1948). Using the information matrix (α_{ij}) with reciprocal (α^{ij}) and the derivatives of log likelihood ϕ_1, ϕ_2, ...all calculated at the hypothetical values of the parameters, a test for the hypothetical values is provided by the χ^2 statistic

$$\Sigma\Sigma\,\alpha^{ij}\phi_i\phi_j$$

with degrees of freedom equal to the number of parameters. When the hypothesis is composite the free parameters have to be estimated and substituted in the above expression which still remains a χ^2 with degrees of freedom lessened by the number of free parameters estimated.

K. S. Rao and Kendall (1950) obtained an elegant proof of the second limit theorem concerning the convergence of the distribution function and its moments.

6.7 *Ordered samples.* A systematic study of ordered samples has been undertaken by Pillai (1948). Out of all ordered statistics the median is seen to contain the maximum information on μ and the least and highest observations on σ. The latter finding brings out the importance of using the mid-range as a simple estimate of σ. The exact distributions of mid-range and semi-range have been developed (1950) in an infinite series. Further work on the use of ordered statistics in tests of significance analogous to the t and F tests is in progress. Patnaik (1950) obtained the distribution of $\bar{x}/\bar{w}$ and $W/\bar{w}$ where W is the range in a given sample and $\bar{w}$ is the mean range from a number of independent samples. He compared the associated power functions with those of t and F.

Nair (1948) gave simple methods of determining the distributions of $u=(x_n-\bar{x})/\sigma$ or $(\bar{x}-x_1)/\sigma$ which is the ratio of the extreme deviation from the

average to the known standard deviation, and also its Studentized form with σ replaced by an independent estimate based on ν degrees of freedom. He gave extensive tables of the probability integral of $u = (x_n - \bar{x})/\sigma$ or $(\bar{x} - x_1)/\sigma$ at intervals of 1/100 for $n = 1$ to 9. The upper and lower 5 p.c. and 1 p.c. significant values of u and its Studentized form are obtained for various values of ν Nair also derived the distribution of the statistic $\delta = \frac{x_n + \ldots x_{n-1+l}}{l} - \frac{x_1 + \ldots + x_k}{k}$ which is the difference between the mean values of the highest l and the lowest k observations and indicated its use in tests of significance. Nair (1947) also compared the relative efficiencies of the mean deviation from the mean and the median.

6.8 *Bivariate surfaces.* Considerable interest was shown in the construction of bivariate surfaces during the last decade. Kibble (1940) derived some bivariate surfaces in a series form having the Gamma type distribution for one and both marginals. Some general classes of bivariate surfaces having different Pearsonian type of curves in the marginals have been obtained by Srivastava (1941). Bivariate correlation surfaces with one and both regressions linear have been constructed by Rao (1942). By considering the joint distribution of the variables $x = u + v_1$ and $y = u + v_2$ where u, v_1 and v_2 are Gamma variates, Cheriyan (1941) derived bivariate surfaces with Gamma distributions in the marginals. By an ingenious device Vaswani (1947) constructed a bivariate surface in which both marginals are normal and the product moment correlation is positive but the regression of y on x is such that y decreases as x increases—a pitfall in correlation theory! Bhattacharya (1945) has investigated into the minimum number of sufficient conditions which lead to a bivariate normal distribution.

6.9 *Problems of Tabulation.* The theoretical distributions of classical and Studentized D^2 statistics are very complicated so that no practical statistician is tempted to use them unless adequate tables are provided. P. K. Bose (1943, 1947) made an extensive study of Bessel functions and developed a simple method of tabulating the percentage points of the classical D^2 statistic. He (1948) has shown how these tables could be utilized in the determination of confidence belts for the theoretical value of D^2 and also in tests of significance.

Following the theoretical reduction of the incomplete probability integral for the Studentized D^2 statistic by P. K. Bose and Roy (1941), P. K. Bose (1949) provided the tables of significant levels of the Studentized D^2 statistic for some values of Δ^2, the population parameter corresponding to D^2.

Chandrasekar and Francis (1941) used an ingenious method to tabulate the 5 p.c. and 1 p.c. significant values of the statistic defined as the ratio of maximum of mean square to sum of all mean squares, each with f degrees of freedom. This is to test whether the highest observed mean square is significantly different from the rest.

Sukhatme (1938a) successfully solved the problem of tabulating 5 p.c. values of Fisher and Behren's test for judging the significance of the difference of the mean in two samples with possibly unequal variances. He also gave the ordinates of the Student's distribution for various values of the degrees of freedom.

Swarup (1938) tabulated the probabilites of the difference in proportions in two independent samples of the same size being equal to and greater than the observed value for a fixed value of the sum of the observed proportions. This is useful in carrying out an exact test of independence in a two-fold contingency table. The sample sizes considered were 5, 10, 15, 20, 30 and 50. He is now preparing more extensive tables.

Nair (1939*a*, 1939*b*) has shown that the distribution of the median in tests by randomization is quite simple and provided extensive tables for deriving the

confidence interval for the median in samples from any continuous population. S. K. Banerji (1936) tabulated 1 p.c. values of the variance ratio and Panse and Aiycht (1944) the 10 p.c. values of the variance ratio F and z for use in plant breeding work.

Patnaik (1949) found approximate methods of calculating the incomplete probability integral of the non-central χ^2 and F distributions which are analogous to the classical and Studentized D^2 statistics. He also considered the use of these probabilities from the point of view of power functions.

7.1 Bhabha (1950) considered a system of particles whose number has a probability distribution and corresponding to a given number k, there is a k dimensional symmetric density function. He has shown how to calculate the mean of various functional averages over the particles, in particular, the expected number and its square of the particles in a given interval. The results are applied to a problem of electron cascades.

7. Estimation

7.1 *Unbiased minimum variance estimate.* It was first shown by R. A. Fisher that the maximum likelihood estimate of a parameter θ has the least limiting variance $1/ni$ where i is the information per single observation defined by

$$i = -E\{d^2 \log p(x/\theta)/d\theta^2\}$$

$p(x/\theta)$ being the probability of a single observation and n the sample size. In a study of this problem it is of some interest to determine a lower limit to the variance attainable in the estimation of a parameter from a sample of any given size. If

(a) T is an unbiased estimate of θ and $V(T)$ denotes the variance of T, and

(b) I is the variance of $d\log P(x/\theta)d\theta$ where $P(x/\theta)$ is the probability density of the observations $x_1,\ldots,x_n$ then, under some general conditions, Rao (1945c) proved that $V(T) \geqslant 1/I$. This provides an easily ascertainable lower limit and when a statistic has $1/I$ as its variance it could be considered as the minimum variance estimate. He (1945c, 1946b) also proved that in the multiparameter case if

(a) $$E\left(\frac{\partial \log P}{\partial \theta_i}, \frac{\partial \log P}{\partial \theta_j}\right) = I_{ij},\ I = (I_{ij})$$

(b) $\psi_1, \ldots, \psi_k$ are k independent functions of q parameters occuring in the probability density.

(c) V is the covariance matrix of $T_1, \ldots, T_k$ the unbiased estimates of $\psi_1, \ldots, \psi_k$ and $\Delta = (\partial\psi_1/\partial\theta_j)$ then the matrix $V - \Delta I^{-1}\Delta'$ where I^{-1} is the inverse of I is positive or semi-definite. Similar results were obtained by H. Cramer working independently in Sweden. Rao (1945c) further proved that the unbiased minimum variance estimates are explicit functions of sufficient statistics and also (1948) any function of the sufficient statistic has the least variance as the unique unbiased estimate of its expected value. A privileged property of sufficient statistics introduced by Fisher is thus established.

The lower limits obtained above are not necessarily attainable in any problem. In fact, examples were provided by Rao (1948) to show that the least variance attainable is not always $1/I$. Bhattacharya (1946, 1947, 1948) improved the above inequalities by introducing the quantities

$$J_{rs} = E\left(\frac{1}{P}\frac{d^r P}{d\theta^r} \cdot \frac{1}{P}\frac{d^s P}{d\theta^s}\right)$$

in which case V(T) the variance of an unbiased estimate of the parameter θ, is not less than J^{11}, the co-factor of J_{11} in (J_{rs}). It may be noted that $J^{11} \geqslant \frac{1}{I}$

He derived a number of other interesting results in the uni and multi-parameter cases. One result of some importance, specially useful in the case of non-continuous parametrs is

$$V(T) \geqslant (\Delta\,\tau)^2 / \int \frac{(\Delta f)^2}{f}\,dv$$

when Δ is the difference operater. This is generalized to the multi-parameter case. The necessary changes when there are constraints on the parameters have also been investigated.

Bhattacharya (1950) further proved that if there exist functions ϕ_1, ϕ_2.....of the parameters and the variables such that $\int T\phi_i\, dv = \alpha_i\ (\theta)$ where T is an unbiased estimate of $\tau(\theta)$ and α_i; (ϕ) is a known function of θ then

$$V(T) \geqslant (\lambda_0 - \tau)\ \tau + \Sigma \lambda_i \alpha_i$$

where λ_i are determinable functions of the variances and covariances of ϕ.

7.2 Seth (1949) extended the results of Bhattacharya to problems in sequential estimation. He showed that the variance of the unbiased sequential estimate of $\gamma(\theta)$ a function of a parameter θ is not less than

$$\Sigma\Sigma\lambda^{ij} \frac{d^i\gamma(\theta)}{d\theta^i}\ \frac{d^j\gamma(\theta)}{d\theta^j}$$

where (λ^{ij}) is the matrix reciprocal to (λ_{ij}) defined by

$$\lambda_{ij} = E\left(\frac{1}{P_N^2}\ \frac{d^iP_N}{d\theta^i}\ \frac{d^jP_N}{d\theta^j}\right)$$

the only difference with the earlier result being that the expectation is taken over the observations as well as the variable sample size. Similar results were proved for the multiparameter case.

7.3 Different methods of constructing interval estimates have been considered by George (1942). A new method called the 'shortest confidence intervals' has been proposed and applied to the interval estimation of the variance of a normal population. A graphical method has also been given for obtaining the shortest confidence intervals in cases where the theoretical distribution is unknown.

Pillai (1946) examined the use of U. S. Nair's transformation of the correlation coefficient in the derivation of confidence intervals and showed that in some respects it gives better results than Fisher's z- transformation.

Huzurbazar (1948) has shown that when the range is independent of the parameter, the consistent solution of the likelihood equation is unique and this is a maximum of the likelihood function with probability 1 as the sample size is increased. For distributions admitting sufficient statistics the likelihood equation is shown (1949) to have a unique soluton which is a maximum of the likelihood function.

A method of estimating parameters in an autoregressive time series was given by Ghurye (1950). Efficiencies of certain linear systematic statistics for estimating dispersion from normal samples were calculated by Nair (1950).

7.4 *Estimation by least squares.* Given a vector of n observations such that $E(x) = \tau A$ and variance-covariance matrix D where τ is a vector of p unknown parameters and A is a known matrix, the problem is to determine a linear function bx' of x such that $E(bx') = m\tau'$ a given function of the unknown parameters and $V\ (bx')$ is the least. In the earlier treatment of the problem it was assumed that $p \leqslant n$ and the rank of $A = p$. Bose (1944) has shown that these two restrictions are unnecessary and gave a method of obtaining the best estimate of $m\tau'$ if it is estimable at all. Bose's method referred to the estimation of any particular parametric function so that the solution could not be put in a simpler form which

necessitated the earlier writers to impose the above restrictions. Rao (1945*a,b*) has shown that even in this case the same procedure of finding the normal equations by the least squares technique and substituting the solutions for τ in m_T' supplied the best estimate of m_T' so that all the restrictions could be withdrawn retaining the well established least square method. He later (1946*a*) extended the work on the testing of linear hypotheses to cover the situations where the inequalities $P \leqslant n$ and/or rank $A=p$ are not true. The cases where the unknown parameters are subject to linear restrictions were also considered. Some intrinsic properties of normal equations have emerged out of these studies. With these extensions the least-square technique is put on solid foundations.

As an extension of least square method Kosambi (1947) proposed to 'estimate' a point $(x_1, \ldots\ldots, x_t)$ from which the sum of squares of perpendicular distances on n surfaces $f_i(x_1 \ldots\ldots x_t)=0$, $i=1, \ldots\ldots n$ is a minimum. He applied this method to a problem in genetics.

8. Testing of Hypothesis

8.1 When the nature of the alternative hypotheses is not known the problem of defining an optimum test presents considerable difficulty except in the special case when a uniformly most powerful test exists. Nandi (1946*a*) suggested the choice of a test which maximises the average power over the set of alternatives equidistant, in some well defined sense from the null-hypothesis. In this theory uniform density of the parameters had been assumed so that the test was not independent of the transformation of parameters. He (1946*b*) also studied the properties of Studentized D^2 statistic from the point of view of the average power.

8.2 A second difficulty that arises in testing of hypothesis is the existence of nuisance parameters. J. Neymen and E. S. Pearson pointed out that the solution depends on the existence of what are called similar regions or regions in the sample space whose size has a given value independent of the unknown parameters under consideration. When the probability density of n observations can be written in the form

$$P(x_1, x_2, \ldots, x_n, \theta, \tau) = \phi_1(F_1, F_2, \ldots, \theta, \tau) \ \phi_2(x_1, x_2, \ldots, \tau)$$

where θ and τ stand for the sets of parameters θ_1, $\theta_2 \ldots$ and τ_1, $\tau_2 \ldots$ and $F_1 = F_i(x_1, x_2, \ldots, \tau)$ Neyman has shown that similar regions exist for the parameters θ_1, $\theta_2 \ldots$ Roy (1947, 1948) extending these results discussed the methods of obtaining uniformly most powerful tests for hypotheses concerning τ with unspecified θ when (i) F_i are independent of τ, (ii) $F_1, F_2 \ldots$ and ϕ_2 are independent of τ, with the additional conditions ensuring that the boundary of the region is independent of θ.

8.3 M. N. Ghosh (1948) showed that when the probability density is of the type

$$P(x_1, x_2, \ldots, \theta_1, \theta_2, \ldots) = G(x)F(\theta)e^{\Sigma g_1(\theta)T_1(x)}$$

and if the functions G, F, g and T obey some regularity conditions, Neyman's method constructing similar regions by choosing portions of the regions of a given relative size ϵ enclosed by the surfaces $T_i, T_i + dT$, $i=1,2,\ldots$ supplies the exhaustive set of similar regions of size ϵ. This is an important result in the further problem of constructing optimum tests of composite hypotheses which do not completely specify the values of all parameters.

8.4 In sequential tests of composite hypotheses. A. Wald approached the problem by defining a suitable weight function and using an appropriate probability ratio test. Nandi (1948) discussed the possibility of setting up a probability ratio test by using the probability densities of the sequence of statistics $[T_n]$. Using the approximate limits suggested by Wald, Nandi proved that the

test terminates in some special cases when T_n is a sufficient estimator of ϕ or the limiting distribution of T_n is normal with variance $0\left(\frac{1}{n}\right)$. The use of such statistics in sequential tests of composite hypotheses has been given. Illustrations for testing a hypothesis concerning σ when the mean is unknown and in general the equality of two sets of parameters were also given.

U. S. Nair (1941) showed that for testing the equality of variances on the basis of estimates v_1 and v_2 on f_1 and f_2 degrees of freedom, an unbiased test is provided by the statistic

$$u^{f_1}(1-u)^{f_2} \text{ where } u = v_1f_1/v_1f_1+v_2f_2)$$

which is equivalent to Neyman-Pearson likelihood criterion

$$L = (v_1^{f_1} v_2^{f_2})^{1/(f_1+f_2)} \div \frac{v_1f_1+v_2f_2}{f_1+f_2}$$

with the sample sizes replaced by the degrees of freedom. This provided sufficient justifications for the current use of the modified criterion.

Other investigations of some importance are the studies of power functions of Student's t for paired samples by Nandi (1947), tests of difference between proportions in a 2×2 table by Patnaik (1948) who also provided approximate tables of power function for given values of p_1, p_2 and the demonstration of unbiased nature of tests of independence in multivariate normal systems by Narain (1949).

8.5 Rao (1950*a*) discussed various logical problems involved in discriminatory analysis. He also gave a general theory of distance power tests analogous to those suggested by P. L. Hsu and A. Wald. An important class of these tests is one which has the property that its power for any given alternative bears a constant ratio to the maximum power possible for that alternative. This procedure seemed to effect a compromise between the alternative methods which favoured either the distant hypotheses (ignoring the neighbourhood of the null hypothesis) and those which favoured the immediate neighbourhood (locally most powerful) hoping that this would be good enough for distant hypotheses also. Roy (1950) considered the problem of discrimination when the apriori probabilities of the various hypotheses were known. Poti (1950) derived the power function of the χ^2 test for comparing the blood group gene frequencies of two samples. This is useful in planning of surveys to collect blood group data for comparative studies. Rao (1950*b*) considered the problem of setting up sequential tests of null hypotheses in cases where the alternatives are unspecified. Abraham Wald reduced this problem to that of testing the null hypothesis against an alternative close to it. This method ensured the best possible discrimination for hypotheses close to the null hypothesis. In the method suggested by Rao a distant hypothesis, when it is true, could be discovered quicker and also the test does not make an explicit use of the loss function. The distribution problem has not been completely solved and the approximate solutions offered by Rao are valid for moderately large samples. Neglecting the excess of the probability ratio over the boundaries, Divatia (1949) derived the sequential probability ratio test for the dispersion in an exponential distribution with a known value of the location parameter.

8.6 One of the problems of great interest is the choice, on the basis of samples from k populations, one or more populations having the maximum values of certain characteristics. For instance one may be asked to choose a population having the greatest mean. Since the decision is to be based on samples it would appear that the best thing to do is to select a certain number of populations which cannot be distinguished on the basis of samples and which possess higher values for the mean than the others. Bahadur (1950) has answered this problem by suitably choosing a risk function in the case of k populations. A more intensive study of

this problem, in the special case of two populations was made by Bahadur and Robins (1950).

9. ANTHROPOMERTY

9.1 Anthropometry is one of those border-line studies where anthropologists and statisticians can fruitfully collaborate. The problem before the statistician is the discrimination of a given number of groups on the basis of a specified set of characters. To him the groups have no other significance except they are characterized by a certain number of measurements. It is the problem of the anthropologist to interpret the significant affinities and differences that emerge from the statistical analysis in terms of the groups as a whole and bring in ethnographic and other evidences to postulate further hypotheses concerning the group (or racial) affinities or inter-relationships and their evolution. Experiments have then to be planned in consultation with the statistician (determining the optimum number and nature of characters to be measured and the method of drawing samples) to collect fresh information and to test the new hypotheses. It is only through this way that unprejudiced research in physical anthropology is made possible. Fortunately, in India the major anthropological projects during the last decade were handled by anthropoligists and statisticians working together. Three such projects were the United Provinces and Bengal anthropometric surveys carried out by Dr. D. N. Majumdar in collaboration with the Indian Statistical Institute and the Maharashtra anthropometric survey conducted by Mrs. I. Karve with the assistance of V. M. Dandekar, Statistician to the Gokhale Institute of Politics and Economics.

9.2 The report on U.P. Anthropometric Survey was published under the joint authorship of Mahalanobis, Majumdar and Rao (1949). The data consisted of 12 characters measured on about 2836 individuals drawn at random from 22 castes and tribes residing in the various districts of the United Provinces. It has been found that the measurements did not strictly conform to the multivariate normal distribution but deviations from normality were not appreciable so that tests of significance and the distance estimates based on the normal distribution were generally valid. The variances and covariances were fairly constant from group to group. In this situation, the differences in the groups could be studied with the help of the distance function D^2. A canonical transformation of the variates showed that the configuration of the mean values of the 22 groups in the eight dimensional character space was confined to a three dimensional subspace so that a three dimensional model gave a fair representation of the mutual distances.

This analysis also revealed that for anthropometric investigations samples of size about 150 for each group are adequate and a few well chosen characters properly utilized, can give a fair idea of the inter-relationships of the various groups. It was also found that indices are not the best discriminators and the usual practice of considering the measurements as well as the indices may lead to misleading results.

The biometric analysis and the ethnographic study led to some interesting anthropological speculations which required further investigation.

9.3 Similar studies were made in the Maharashtra anthropometric survey by Karve and Dandekar (1951). The material consisted of 17 measurements on about 2252 individuals belonging to 58 different castes. An excursion into the pitfalls of the statistical and anthropological methods and the roles to be played by the statistician and the anthropologist is an important feature of this report. The report on Bengal anthropometric survey is being written up and will soon be published. Majumdar also conducted an anthropometric survey of Gujrat, the report of which appeared in the Journal of the Gujrat Research Society. Earlier, Mahalanobis (1927) made an extensive study of the Bengal castes and tribes using

test terminates in some special cases when T_n is a sufficient estimator of ϕ or the limiting distribution of T_n is normal with variance $0\left(\frac{1}{n}\right)$. The use of such statistics in sequential tests of composite hypotheses has been given. Illustrations for testing a hypothesis concerning σ when the mean is unknown and in general the equality of two sets of parameters were also given.

U. S. Nair (1941) showed that for testing the equality of variances on the basis of estimates v_1 and v_2 on f_1 and f_2 degrees of freedom, an unbiased test is provided by the statistic

$$u^{f_1}(1-u)^{f_2} \text{ where } u = v_1 f_1 / v_1 f_1 + v_2 f_2)$$

which is equivalent to Neyman-Pearson likelihood criterion

$$L = (v_1^{\ f_1} v_2^{\ f_2})^{\ 1/(f_1+f_2)} \div \frac{v_1 f_1 + v_2 f_2}{f_1 + f_2}$$

with the sample sizes replaced by the degrees of freedom. This provided sufficient justifications for the current use of the modified criterion.

Other investigations of some importance are the studies of power functions of Student's t for paired samples by Nandi (1947), tests of difference between proportions in a 2×2 table by Patnaik (1948) who also provided approximate tables of power function for given values of p_1, p_2 and the demonstration of unbiased nature of tests of independence in multivariate normal systems by Narain (1949).

8.5 Rao (1950*a*) discussed various logical problems involved in discriminatory analysis. He also gave a general theory of distance power tests analogous to those suggested by P. L. Hsu and A. Wald. An important class of these tests is one which has the property that its power for any given alternative bears a constant ratio to the maximum power possible for that alternative. This procedure seemed to effect a compromise between the alternative methods which favoured either the distant hypotheses (ignoring the neighbourhood of the null hypothesis) and those which favoured the immediate neighbourhood (locally most powerful) hoping that this would be good enough for distant hypotheses also. Roy (1950) considered the problem of discrimination when the apriori probabilities of the various hypotheses were known. Poti (1950) derived the power function of the χ^2 test for comparing the blood group gene frequencies of two samples. This is useful in planning of surveys to collect blood group data for comparative studies. Rao (1950*b*) considered the problem of setting up sequential tests of null hypotheses in cases where the alternatives are unspecified. Abraham Wald reduced this problem to that of testing the null hypothesis against an alternative close to it. This method ensured the best possible discrimination for hypotheses close to the null hypothesis. In the method suggested by Rao a distant hypothesis, when it is true, could be discovered quicker and also the test does not make an explicit use of the loss function. The distribution problem has not been completely solved and the approximate solutions offered by Rao are valid for moderately large samples. Neglecting the excess of the probability ratio over the boundaries, Divatia (1949) derived the sequential probability ratio test for the dispersion in an exponential distribution with a known value of the location parameter.

8.6 One of the problems of great interest is the choice, on the basis of samples from k populations, one or more populations having the maximum values of certain characteristics. For instance one may be asked to choose a population having the greatest mean. Since the decision is to be based on samples it would appear that the best thing to do is to select a certain number of populations which cannot be distinguished on the basis of samples and which possess higher values for the mean than the others. Bahadur (1950) has answered this problem by suitably choosing a risk function in the case of k populations. A more intensive study of

this problem, in the special case of two populations was made by Bahadur and Robins (1950).

9. Anthropomerty

9.1 Anthropometry is one of those border-line studies where anthropologists and statisticians can fruitfully collaborate. The problem before the statistician is the discrimination of a given number of groups on the basis of a specified set of characters. To him the groups have no other significance except they are characterized by a certain number of measurements. It is the problem of the anthropologist to interpret the significant affinities and differences that emerge from the statistical analysis in terms of the groups as a whole and bring in ethnographic and other evidences to postulate further hypotheses concerning the group (or racial) affinities or inter-relationships and their evolution. Experiments have then to be planned in consultation with the statistician (determining the optimum number and nature of characters to be measured and the method of drawing samples) to collect fresh information and to test the new hypotheses. It is only through this way that unprejudiced research in physical anthropology is made possible. Fortunately, in India the major anthropological projects during the last decade were handled by anthropoligists and statisticians working together. Three such projects were the United Provinces and Bengal anthropometric surveys carried out by Dr. D. N. Majumdar in collaboration with the Indian Statistical Institute and the Maharashtra anthropometric survey conducted by Mrs. I. Karve with the assistance of V. M. Dandekar, Statistician to the Gokhale Institute of Politics and Economics.

9.2 The report on U.P. Anthropometric Survey was published under the joint authorship of Mahalanobis, Majumdar and Rao (1949). The data consisted of 12 characters measured on about 2836 individuals drawn at random from 22 castes and tribes residing in the various districts of the United Provinces. It has been found that the measurements did not strictly conform to the multivariate normal distribution but deviations from normality were not appreciable so that tests of significance and the distance estimates based on the normal distribution were generally valid. The variances and covariances were fairly constant from group to group. In this situation, the differences in the groups could be studied with the help of the distance function D^2. A canonical transformation of the variates showed that the configuration of the mean values of the 22 groups in the eight dimensional character space was confined to a three dimensional subspace so that a three dimensional model gave a fair representation of the mutual distances.

This analysis also revealed that for anthropometric investigations samples of size about 150 for each group are adequate and a few well chosen characters properly utilized, can give a fair idea of the inter-relationships of the various groups. It was also found that indices are not the best discriminators and the usual practice of considering the measurements as well as the indices may lead to misleading results.

The biometric analysis and the ethnographic study led to some interesting anthropological speculations which required further investigation.

9.3 Similar studies were made in the Maharashtra anthropometric survey by Karve and Dandekar (1951). The material consisted of 17 measurements on about 2252 individuals belonging to 58 different castes. An excursion into the pitfalls of the statistical and anthropological methods and the roles to be played by the statistician and the anthropologist is an important feature of this report. The report on Bengal anthropometric survey is being written up and will soon be published. Majumdar also conducted an anthropometric survey of Gujrat, the report of which appeared in the Journal of the Gujrat Research Society. Earlier, Mahalanobis (1927) made an extensive study of the Bengal castes and tribes using

the generalized distance D^2 devised by him. The material used for this purpose consisted of a long series of measurements taken by the well known anthropologist N. Annandale on Anglo-Indians (persons of mixed European, usually British, and Indian parentage) in Calcutta.

9.4 Indian anthropologists and statisticians have been always stressing the need for securing comparable data from various parts of India. Investigator bias and lack of standardization in the measurements of some somatic characters reduce the validity of comparisons (Mahalanobis, 1930; Chattopadyay, 1942; and Majumdar, 1940). Under laboratory conditions Chattopadyay (1949) found that different investigators trained in the same laboratory are not likely to differ much. But how far this is true under field conditions requires further investigation. Mahalanobis (1937) found that measurements on the profile recorded on photographic paper will go a long way in reducing the labour on the field and also in securing comparability as in the case of measurements on the skulls. Further research in this direction to obtain photographs in different angles and to reconstruct from them some of the direct measurements on the head is in progress.

9.5 Anthropometric measurements in India were first systemetically taken by Herbert Risley and published as a volume of Ethonographic Appendices to the Census report of India, 1891. In the United Provinces Drake—Brockman undertook an extensive anthropometric survey, the results of which were published in the first volume of *Castes and tribes of North Western Provinces and Oudh,* 1896. As a supplement to Brockman's note, tables of measurements carried out under the supervision of E. J. Kitts were published by Crooke. In 1931 B. S. Guha undertook a racial survey of India and measured about 2511 individuals. The statistical analysis, where in the coefficient of racial likeness is used for determining racial affinities and divergences, is published as Part III of the *Census of India,* 1951, Vol. 1. All these earlier investigations suffered from one defect or the other. Either samples were not drawn strictly at random or the sample sizes were inadequate for drawing any reliable conclusions.

10. Population Studies and vital Statistics

1.1 It is said that the vital statistics in any country ought to be the main platform for population discussions, especially when the growth of population, as distinct from the quantum at one time, is in question. Absence of reliable vital statistics greatly retarded the progress of population studies in India. No steps have been taken to improve the position since the Bhore Committee recommended the establishment of a suitable machinery to ensure the reliability of the collected statistics of births, deaths and the causes of death.

The extensive study of Chandrasekar (1949*a*) of the Parsi demography has shown that useful inferences can be drawn from an analysis of vital statistics which, in the case of the Parsi community were found to be fairly reliable. Some theoretical investigations which arose during this study, to determine the effect of a change in mortality conditions in an age group on the expectation of life at birth formed the subject matter of another paper by Chandrasekar (1949*b*).

The main defect in our vital statistics appears to be 'appreciable omission of registered births, deaths and cases of notifiable diseases'. Attempts have been made to correct these figures by independent enquiries in certain localities. An instance is the Singur health survey where the figures supplied by the investigators have been used to correct the number of births registered (Chandrasekar and Deming, 1949). One assumption was made that the chance of a registered or an unregistered birth coming into the investigator's sample is the same. This may not be strictly true but the method employed is an interesting one and will certainly be useful in similar studies.

Some type studies of demographic conditions of rural population were made by Mathen and Lal (1946), Chandrasekhar and Sen (1948), Sovani (1948) and others on the basis of a detailed survey of a few villages. It is only by such studies in a large number of villages scattered all over the country that one can hope to obtain estimates of specific fertility rates and death rates which are necessary for population projection.

10.2 The first step towards filling the gap in vital statistics was made when the census commissioner M. W. M. Yeatts extended the questionnaire of the 1941 census to cover some items of information useful in the calculation of specific fertility rates, net reproduction rates and life tables. Unfortunately, the information carefully collected on individual slips could not be fully tabulated due to prevailing war conditions immediately after the 1941 census. When the destruction of the record slips was under contemplation the census commissioner thought it fit to preserve at least every fiftieth slip and what is left today out of about 39 crores of slips is only a sample of one in fifty, now called the *Y* sample. The Government of India appointed a Committee with Yeatts as chairman. Mahalanobis, Madhava, Raja and Gregory as members to examine and advise the Government on the available data relating to growth of population and the *Y* sample formed the key to the Committee's operations. Their report (1945) published by the Department of Education, Health and Lands, contains extensive studies on tests of representativeness of the *Y* sample and the possibilities of constructing the life tables, net reproduction rates etc. Tabulation of the *Y* sample along these lines is in progress in the Indian Statistical Institute. The Committee also recommended the preparation of a house list, first in Bengal and then all over India, to be used as frame for population and socio-economic surveys. When such a list is ready, population estimates and detailed vital statistics can be obtained by sample enquiries at a very low cost.

11. Economic Statistics

11.1 Progress in economic statistics during the last decade is mainly confined to the family budget studies and the construction of the index of business activity. In a preliminary study of the working class family budgets Sinha (1940) found that the preference scale is linear in the case of consumer goods so that Engel's law holds good. He gave an illustration of the various computations leading to the construction of the cost of living index number. The approach is atomistic. Similar work was done by a number of other workers.

Poti (1946) found that variations in the family expenditure patterns could be sufficiently explained by a single factor obtained as a linear combination of the proportions of income spent on various goods. He proposed to use this as an index to classify the families according to consumption patterns. This is an important investigation since methods of classification by income or social group seem to have no economic significance unless the class intervals are wide apart.

11.2 For several years there was no satisfactory index of business activity or industrial production or even of agricultural production. Earlier attempts made by Findlay Shirras left much to be desired. Meek (1937) and S. R. Bose (1837) carried the work further by a suitable choice of the material and formulæ for the construction of index numbers. A more elaborate index with proper weights was constructed by 'Capital' and published for the first time in 1938. A fairly satisfactory index of business activity is due to Sinha and Roy (1941). They carried out appropriate corrections for seasonal and cyclical fluctuations in the various series and derived suitable weights by using the correlation coefficients between different series. Following the method suggested by Rhodes, Sastry (1941) constructed indices of industrial productivity in India.

At present the most satisfactory index is published in the 'Monthly statistics of the production of selected industries in India'. The weights are derived from the census of production and manufacture and hence are more accurate than those hitherto used. The items covered are large in number so that all types of business activity are represented. Appropriate corrections for seasonal and cyclical fluctuations and abnormal features, such as the effect of war, have been made.

11.3 Other studies of interest are by Mahalanobis (1941) who developed a new formula for estimating the rupee coins in circulation assuming a simple law of constant rate of absorption. Murti (1947*a*) fitted a logistic curve to the average life of a rupee coin and estimated the circulation of the rupee notes for various years.

Using four variables, Murti (1947*b*) deduced an index by the method of principal components to classify the scheduled banks into some distinct groups each having a common trait.

11.4 Trends of foreign trade have been studied by Sastry and Mathew (1940) and others. A comprehensive study of the consumer's expenditure as a sector of the national income of India was made by Desai (1948). For the first time estimates of errors in the various aggregates have been provided. Sinha (1939) studied the inter-relationship between supply and price of raw jute. Analysis based on Schultz's method of constructing demand and supply functions was carried out by various workers and significant conclusions were obtained.

11.5 In two important articles Madhava (1941) and with Krishna Sastry (1941) studied the growth of trade unions and the problems of absenteeism in Indian Labour.

11.6 Ambica Ghosh (1950*a*, 1950*b*) applied statistical techniques in an analysis of the agricultural economy in Bengal. Using the concentration curves of owned land in different ranges of ownership and the concentration of holding cultivated in these ranges of ownership and the concentration of holding cultivated in these ranges as cumulative percentage of area owned under different acreage intervals and the corresponding cumulative percentage of area cultivated, Ghosh classified different districts of Bengal into 4 regions with more homogeneous economic features. The two variables considered above correspond to the concentration of the *volume of production* and the concentration of the *means of production* respectively. He also found 2 acres as the marginal holding and 7 as the optimum size of a holding in Bengal.

11.7 A large number of socio-economic surveys were conducted by the Gokhale Institute of Politics and Economics. A survey of farm business in Wai Taluka by Gadgil and Gadgil (1940) supplied the basic information for reviewing the farming practices and advising the farmer to improve the economic organization of his farm. Other surveys of interest include marketing of fruit in Poona (Gadgil and Gadgil, 1933), Motor bus transportation (Gadgil and Gogate, 1935), socio-economic conditions of weaving communities in Sholapur (Kakade, 1947), economic effects of irrigation (Gadgil, 1948), and investigtiaon into the problem of lapse into illiteracy in Satara District.

12. Education and Psychology

12.1 Considerable interest was shown in the application of statistical methods in educational and psychological problems. Early in 1928 intelligence tests through the medium of Bengali were devised and administered to over 4000 Bengali boys. The material was analyzed by Mahalanobis in a series of papers (1933, 1934) and the validity of these tests was established. The intelligence scores showed significant and high correlations with success in school examinations.

Hussain (1940, 1941) considered some methods of standardizing examination marks. While it is recognised that the standard of examinations should be maintained at some level, the method of adjusting the marks by way of correction for a high or low standard is not universally accepted. Hussain recommended the method of individual ranks as a reasonable one but sufficient justification for such recommendation was not provided.

In an interesting experiment on rats, Kuppuswamy (1941) found that training given to parents had no significant effect on the progeny, indicating that acquired habits are not likely to be inherited.

Krishnaswamy Ayyanger (1941) studied the different types of trilinear signature of men (three oriented lines which they are asked to draw) and conjectured that these configurations refer to the psychological traits of persons.

12.2. An important contribution to psychological tests is due to Amiya Sen (1950) who studied the validity of Roschah inkblot tests in revealing different personality types. Using 100 Indian students (60 men and 40 women) as subjects she found that many of the subjective interpretations of the inkblot studies were contradictory to what was revealed by an objective factor analysis. Finally these tests were shown to have greater validity when scored by the method suggested by Burt.

13. Genetics

13.1 *Plant breeding.* Three essential requirements of a plant breeding programme are (a) choice of material for selection, (b) procedure of selection and (c) multiplication and maintenance of strains. Among materials from different sources, differing in mean values and genetic variability, the expected response to a given degree of selection can be calculated by assuming a normal distribution of genetic values and this can be used as the criterion in the choice of the most suitable material for selection. An instance of such calculation is given by Panse (1947). This needs the knowledge of the genetic variability which can be estimated from the data of the replicated progeny trial with seed from a number of plants selected randomly from plant material under examination. One method of estimation is to find the true variance between progenies on progeny basis and the other is to take the regression of progeny means on the values of the parental plants and find the genetic variance of the parental population by means of the formula: genetic variance $= Vb$; where b stands for the regression coefficient and V the gross variance of the parental population. The theory of this was given by Panse (1940) when no dominance is involved and by Panse and Bokil (1948) when the effect of dominance is considered.

13.2 Replicated progeny row trial again forms the basic method for exercising selection in the chosen material. Selection to be efficient should be made primarily on the basis of progeny means, and then the selected progeny should be propogated through single plants selected within these progenies. There is difficulty in the choice of selecting single plants with higher genotypic values. Panse (1940) has shown that selection of single plants on the basis of deviations of individual plants from the means of the plot to which they belong reduces the chance of the selection being unduly influenced by favourable or unfavourable environment. The method of discriminant function developed by H. Fairfield Smith is now extensively used for this purpose. The discriminant function is a suitability chosen linear function of the characters aimed at maximizing the genetic advance. Nanda (1949*a*) found the standard errors of the discriminant function coefficients and the genetic advance. Nanda (1949*b*) also illustrated the computations of genetic advance and its standard error. The formula (15) in his paper connecting the genetic advances by straight selection and the discriminant function needs some correction.

Panse (1940) also considered some genetic models in studying the quantitative inheritance of characters and provided expressions for determining the 'effective' number of factors which can account for the segregation in F_2 generation.

13.3. *Estimating of linkage.* Some work has been done in methods of scoring linkage data from various sources for purposes of estimating recombination fractions by Bhat (1948). Kosambi (1944) demonstrated that the recombination fraction y_1, y_2 for two individual adjacent segments and the fraction y_3 for the whole segment are connected by the approximate relation

$$y_3 = \frac{y_1 + y_2}{1 + 4y_1y_2}$$

Sukhatme (1941) obtained standard errors for Bernstein's estimates of blood group gene frequencies. Rao (1950) provided the complete scoring system for the simultaneous estimation of recombination fractions from three point data giving the simultaneous segregation of three factors.

14. Quality Control

14.1 Modern statistical techniques have a great part to play in Industrial development in India. If Indian goods have to enter into a competetive market they must be produced according to a 'specification' and as cheaply as possible. At present many of the factory practices are wasteful being based on assumptions and beliefs often handed down from generation to generation by tradition. Systematic collection of data would enable the technicians to test the validity of such traditional methods and to improve or improvise new methods when the earlier ones are not found to be effective. For example, it is an established practice in any factory to make frequent adjustments on the machines rather than try to locate actual defects. The control chart method would be of great help in locating the assignable causes of variation and also in showing what variations should be ignored and what variations need a corrective action.

14.2 The Indian industrialists have been slow in adopting the newly evolved techniques and it is the responsibility of the Indian Statisticians to sell their goods. The Indian Society for Quality Control, founded in 1948, is doing good service in arousing the interests of the Indian Industrialists in the usefulness of modern statistical techniques. Their first bulletin written by Chameli Bose contains the uses of quality control techniques in design, production and inspection. More bulletins of this nature with illustrative examples will be useful in convincing the producers of the advantages of applying statistical techniques in production.

14.3 At present there is a statistical division in the Tata Iron and Steel Works at Jamshedpur organised by A. V. Sukhatme. They answer a variety of problems arising out of production, inspection and distribution of finished goods.

14.4 An outstanding achievement in the field of statistical Quality Control is due to the statistical division in the Ahmedabad Textile Institute Research Association (ATIRA) under the leadership of Sundri Vaswani. In five member mills where statistical quality control work was introduced, without exception, it resulted in better maintenance of machinery, reduction in waste, improvement in quality, reduction in work-loads, increase in output and reduction in machine stoppages (Sundri Vaswani, 1949, 1950). Now there is a comprehensive plan to introduce similar methods in all the 71 member mills of the ATIRA.

14.5 Some important comments were made by Pran Nath (1948) on sampling inspection plans suggested by E. L. Grant who thought that an increase in sample size increases the effectiveness of sampling inspection. Nair (1949) discussed the possibility of applying quality control methods in wood based industries.

15. Meteorology

15.1 The present state of meteorological prediction has been summarised by the editor of a daily newspaper as follows:

> "Before the meteorologists are dismissed as prophets of the 'Utterly absurd' as they might well be since recent Calcutta performances, it should be considered that theirs is not an exact science, but chiefly descriptive. Given sufficient data, frequently supplied and adequately correlated, they may, if pressed, draw inferences whose reliability will vary according to the strength of the indications and in inverse ratio with the duration of the period to which these relate. That may be enough about meteorologists".

No doubt the meteorological statistician is working in a difficult field fighting the vaguaries of weather. Some of the results which are the product of imagination and intensive research are highly interesting.

The prediction of monsoon rainfall is very important for an agricultural country like India. During the researches carried out towards the end of the last century, it was discovered that preceding conditions outside India must play a large part in determining monsoon rainfall. Sir Gilbert Walker working from 1904 provided a prediction formula by computing the regression line of best fit of the monsoon rainfall with some of the preceding weather conditions outside India. A typical example of the regression equation is the 1910 formula.

$$\begin{aligned}\text{Monsoon Rainfall} = &-.20\ (\text{Snowfall Accumulation}) \quad -.29\ (\text{Mauritius Pressure}) \\ &+.28\ (\text{South American Pressure}) -.12\ (\text{Zanzibar Rainfall})\end{aligned}$$

for forecasting rainfall in India as a whole. This was the earliest use of a regression formula found to work well in practice.

15.2 A few years later different formulæ were constructed for different regions and these formulæ are being constantly revised. Details of different methods of 'foreshadowing' monsoon and winter rainfall are given in paper by S. K. Banerji (1950)..

The construction of the prediction formula involved the selection of a few characters out of large number and the success of such a formula can be judged only by its performance, i.e., 'waiting and seeing its behaviour'. The predicted values and the subsequently observed values must agree closely and the tests of agreement developed by Normand (1932) and extended by Savur (1932) to suit various applications such as testing for the reality of periods in periodogram analysis etc., are very useful devices in this connexion.

15.3 A number of useful investigations on the meteorological data relating to rainfall, river floods, temperature and other aspects have been carried out during the last decade and these are published in Memoirs of the Indian Meteorological Society and also in the Scientific Notes of the Indian Meteorological Department.

16. Miscellaneous

16.1 Reference may also be made to a number of interesting investigations undertaken by authors who, it may be said, 'did not so much desire to find truth as to cure their mental itch'. But in these may be found seeds for future development. Sometimes the investigations proved mathematically difficult, for instance the frequency of digits in π, $\sqrt{2}$, etc., and sometimes laborious as in measuring the length of life line on human palms or throwing coins a large number of times. The conclusions were as interesting as the investigations themselves.

16.2 One author thought that Neyman Pearson's theory of testing of hypothesis is a synthesis of the errors in east and west, the error of the first kind

having been recorded by *Kalidasa* in the east when *Dushyanta* rejected a true hypothesis in not receiving his lawfully wedded wife *Sakuntala* and the second kind of error by Shakespeare in the West when Othello accepted a false hypothesis that Desdemona committed adultery. The problem arises then as to which error should be minimized !

16.3 A good deal of interest was shown in a statistical study of literary styles of various authors. Some work done on the writings of the Indian poet Rabindranath Tagore has revealed a few interesting aspects of the chronological changes in style. Kosambi (1942) disproved the hypothesis advanced by the philologist Zipf that the number of words used n times in a connected piece is proportional to $1/n^2$.

16.4 Rao and Shaw (1948) found a new formula for the prediction of cranial capacity and demonstrated that the skulls which are preserved in tact and admit a direct measurement of the capacity are on the whole undersized so that the average cranial capacity so determined is an underestimate of the capacity of the whole cranial population. They have also given a method by which the capacity could be estimated from the measurements of the other fragments and thus correct for the skulls providing direct measurement of the cranial capacity. If the suggested hypothesis, that on the whole small skulls are preserved intact for a longer time, is true then there is reason to believe that our human ancestor had a much bigger skull than what has been revealed by the fossil skulls.

16.5 On the theoretical side Singh (1946) developed an interesting theory of quasi-limit definition of probability. After demonstrating that Von Mises' limit introduces some regularity in the so called 'irregular collective', Singh tried to modify the limit as follows. If S_n denotes the relative frequency up to n terms of the sequence then the existence of the quasi-limit p implies that given two positive numbers ϵ and η as small as we please, two numbers n_0 and m_0 can be found such that of the m terms $S_{n+1}, \ldots, S_{n+m}$, π terms satisfy the inequality $|S_{n+i_1} - p| < \epsilon$ where $\pi/m > 1 - \eta$ whenever $n \geqslant n_0$ and $m \geqslant m_0$. This again introduces some regularity and it is recently shown by D. Basu that quasi-limit implies the existence of Von Mises' limit. But the approach seemed novel.

16.6 An author was led to demonstrate that enforced widow-hood is the only cause of slower growth of Bengali Hindus. The zeal with which the figures were presented without any cautious attempt to interpret them properly reveals that the author was extremely interested in this problem.

16.7 An early statistical venture is the estimation by Mahalanobis (1937) of the expected increase in yield of paddy by the introduction of an irrigation scheme by which Damodar flood was intended to be used to supplement the rainfall. In this study the water requirement for a full crop and the method of adjustment were presented without any cautious attempt to interpet them properly reveals occured were first obtained as an agreed verdict by five experienced agricultural officers. Using the rainfall data for any year the yield was evaluated as percentage of full crop and then the increase in yield by using the available Damodar flood was obtained.

This report by Mahalanobis gave support to the Damodar valley project since developed by the Government of India.

16.8 In a series of papers Koshal (1933, 1935, with Turner, 1930) applied the method of maximum likelihood in estimating the constants of Pearsonian curves. First approximate values were obtained by the method of moments and then by a series of approximations the maximum likelihood estimates were approached. These papers gave rise to bitter controversies between Karl Pearson and R. A. Fisher, the former defending the method of moments and the latter pointing out its ineffectiveness in providing suitable estimates. For the first time the difficulties

encountered in fitting Pearsonian curves by the method of moments have come to be discussed in great detail.

16.9 Unconventional methods are as important as properly codified scientific ones in any branch of science. One such instance in statistics is the method employed by three statisticians (S. B. Sen, J. M. Sen Gupta and D. M. Ganguli deputed by the Indian Statistical Institute at the request of the Central Goernvment) in estimating the number of Muslim refugees in the red fort after the great riot in 1947. Any attempt at a personal examination of the spot and obtaining sample counts of individuals at random localities inside the fort would have meant the complete anhilation of the statisticians who happened to be Hindus. They ascertained the amount of salt (a commodity not likely to be drawn in excess of requirement) issued to the refugees from the Government ration shops and dividing this amount by the average salt consumption which is almost stable in any group of individuals they estimated the number of refugees. This estimate was useful to the Government in arriving at some administrative decisions. This method, to give it an indigenous name, may be called the *nimmak* method.

BIBLIOGRAPHY

1. *Introduction*

Mahalanobis, P. C. (1923). On errors of observation and upper air relationship. *Mem. Ind. Met. Dept.*, **24**, 10.

Wold, H. (1938). A Study in the Analysis of Stationary Time Series, Uppsala.

2. *Official Surveys*

Subramaniam, S. (1947). Recent developments in Indian official statistics. *Proc. Int. Statist. Conf.* **2**, 193.

3. *Sample Surveys*

Banerji, S. K. *et. al.* (1948). Studies in sampling technique : 1. Estimation of whitefly (Aierrolobus Barodensis) incidence in sugarcane. *Proc. Ind. Acad. Sci.*, **28**, 107.

Bose, C. (1943). Note on the sampling error in the method of duble sampling. *Sankhya*, **6**, 329.

Chakravarty, N. C. (1937). Preliminary Report of the Survey of Handloom Weaving Industry in Bengal. Published by Govt. of Bengal.

Das, A. C. (1950). Two dimensional systematic sampling and the associated stratified and random sampling. *Sankhya*, **10**, 95.

Ghosh, B. N. (1943*a*). On the distribution of random distances in a rectangle. *Sci. and Cult.* **8**, 388.

——— (1943*b*). On some artificial fields and their correlation function. *Ibid.*, **8**, 424.

——— (1943*c*). On random distances between rectangles *Ibid.*, **8**, 464.

——— (1943*d*). On sampling in unknown fields. *Ibid.*, **9**, 129.

——— (1943*e*). On the construction of some natural fields. *Ibid.*, **9**, 213.

——— (1947*a*). Double sampling with many auxiliary variates. *Bull. Cal. Stat. Assoc.*, **1**, 91.

——— (1947*b*). Representative population set of points in multi-dimensional fields. *Proc. Ind. Sci. Cong. Part III*, 34th Session, 10.

——— (1949*a*). Variance and covariance of rectangular and linear sample-units for a type of natural fields. *Proc. Ind. Sci. Cong. Part III*, 36th Session, 8.

——— (1949*b*). Topographic variations in statistical fields, *Bull. Cal. Stat. Assoc.*, **2**, 65.

——— (1949*c*). Topographic randomness and its statistical tests, *Ibid.*, **2**, 65.

——— (1949*d*). Interpenetrating net works of samples. *Ibid.*, **2**, 108.

——— (1950). A method of topographic analysis for a type of weighted variate. *Ibid.*, **3**, 53.

Hubback, J. A. (1927). Sampling for rice yield in Bihar and Orissa. Bulletin No. 166, Agricultural Research Institute, Pusa. Reprinted in *Sankhya*, **7**, 281.

Mahalanobis, P. C. (1937). Statistical note on the Hooghly-Howrah Flushing and Irrigation Scheme. Report to Govt. of Bengal, Irrigation Dept.

——— (1939). Report of the census of Jute in 1938. Published by the Indian Central Jute Committee.

——— (1941*a*). Report on the programme preference and broadcast reaction. (Submitted to the Department of Information, Govt. of India).

——— (1941*b*). Report on the sampling technique for forecasting the bark yield of cinchona plants. (Submitted to Superintendent, Cinchona Culture in Bengal).

——— (1942). Family budget enquiries of labourers, Jagaddal. (Published by the Board of Economic Enquiry, Bengal).
——— (1943). An enquiry into the prevalence of drinking tea among middle class Indian families in Calcutta. *Sankhya*, **6**, 283.
——— (1944). On large scale sample surveys. *Roy. Soc. Philos. Trans.* B, **231**, 329.
——— (1945). Report on the Bihar Crop Survey : Rabi season 1943-44. *Sankhya*, **7**, 29.
——— (1946*a*). Sampling surveys of crop yields in India. *Sankhya*, **7**, 269.
——— (1946*b*). A multipurpose sampling scheme for Bengal. (Report submitted to the Govt. of Bengal).
——— (1946*c*). Use of small size plots in sample surveys for crop yield. *Nature*, **158**, 798.
Mahalanobis, P. C., Mukherjee, R. K. and Ghosh, A. (1946). A sample survey of after-effects of the Bengal famine of 1943. *Sankhya*, **7**, 337.
Nair, K. R. (1950). Sampling techniques. *Indian Forester*, **76**, 31.
Panse, V. G. and Kalamkar, R. J. (1945). A large scale yield survey on cotton. *Current Science*, **14**, 287.
Sukhatme, P. V. (1945). An interim report of the crop-cutting survey for estimating the outturn of wheat in the Punjab (1943-44). *Proc. Ind. Acad. Sci.*, **21**, 328.
——— (1946*a*). Bias in the use of small-size plots in sample surveys for yields. *Nature*, **157**, 630.
——— (1946*b*). Bias in the use of small-size plots in sample surveys for yield. *Current Science*, **15**, 119.
——— (1946*c*). Size of sampling unit in yield surveys. *Nature*, **158**, 345.
——— (1947*a*). Use of small size plots in yield surveys. *Ibid.*, **160**, 542.
——— (1947*b*). The problem of plot size in large-scale yield surveys. *J. Am. Stat. Assoc.*, **42**, 297.
——— (1950). Efficiency of subsampling designs in yield surveys. *J. Ind. Soc. Agri. Stat.*, **2**, 212.
Sukhatme, P. V. & Panse, V. G. (1948). Crop surveys in India. *J. Ind. Soc. Agri. Stat.*, **1**, 34.

4. *Experimental designs*

Banerji, K. S. (1948). Weighing designs and balanced incomplete blocks. *Ann. Math. Stat.*, **19**, 394.
——— (1949*a*). A note on weighing design. *Ann. Math. Stat.*, **20**, 300.
——— (1949*b*). On certain aspects of spring balance designs. *Sankhya*, **9**, 367.
Bhattacharya, A. (1942). A note on Ramamurti's problem of maximal sets. *Sankhya*, **6**, 189.
Bhattacharya, K. N. (1944). On a new symmetrical balanced incomplete block design. *Bull. Cal. Math. Soc.*, **36**, 91.
——— (1946). A new solution in symmetrical incomplete block designs *Sankhya*, **7**, 423.
——— (1950). Problems in partially balanced incomplete block designs. *Bull. Cal. Stat. Assoc.*, **2**, 177.
Bose, S. S. & Mahalanobis, P. C. (1935). On estimating individual yields in the case of mixed up yields of two or more plots in agricultural field experiments. *Science & Culture*, **1**, 205.
Bose, R. C. (1939). On the construction of incomplete block designs. *Ann. Eugenics.* London, **9**, 358.
——— (1942). An affine analogue of Singer's theorem. *J. Ind. Math. Soc.*, **6**, 1.
——— (1947). Mathematical theory of the symmetrical factorial design. *Sankhya*, **8**, 107.
Bose, R. C. & Nair, K. R. (1939). Partially balanced incomplete block designs. *Sankhya*, **4**, 337.
Bose, R. C. & Kishen, K. (1940). On the problem of confounding in the general symmetrical factorial design. *Sankhya*, **5**, 21.
Kishen, K. (1941). Symmetrical and unequal block arrangements. *Sankhya*, **5**, 323.
——— (1942). On expressing any single degree of freedom for treatments in an s^m factorial arrangement in terms of its sets for main effects and interaction. *Sankhya*, **6**, 133.
——— (1945). On Hotelling's weighing problem. *Ann. Math. Stat.* **16**, 294.
——— (1947). On fractional replication of the general symmetrical factorial design. *Current Science*, **16**, 138.
——— (1949). On the construction of Latin and Hyper-Graeco-Latin Cubes and hyper-cubes. *J. Ind. Soc. Agri. Stat.*, **2**, 20.
——— (1950). Expression of unitary components of higher order interactions in 3^5, 3^6, 4^4 and 5^4 designs in terms of sets for these interactions. *J. Ind. Soc. Agri. Stat.*, **2**, 196.
Mahalanobis, P. C. & Bose, S. S. (1933). An analysis of manurial experiments on wheat conducted at Sakrand, Sind. *Ind. Jour. Agri. Sci.*, **3**, 345.
Nair, K. R. (1938). On a method of getting confounded arrangements in the general symmetrical type of experiment. *Sankhya*, **4**, 121.
——— (1939). The application of covariance technique to field experiments with missing or mixed up plots. *Sankhya*, **4**, 581.

——— (1944*a*). The recovery of inter-block information in incomplete block designs. *Sankhya*, **6**, 383.
——— (1944*b*). Calculation of standard errors and tests of significance of different types of treatment comparisons in split-plot and strip arrangements of field experiments. *Ind. J. Agri. Sci.*, **14**, 315.
——— (1950). Partially balanced incomplete block designs involving only two replications. *Bull. Cal. Stat. Assoc.*, **3**, 83.
Nair, K. R. & Rao, C. R. (1942*a*). A note on partially balanced incomplete block designs. *Science & Culture*, **7**, 516.
——— (1942*b*). Incomplete block designs for experiments involving several groups of varieties. *Science & Culture*, **7**, 625.
——— (1948). Confounding in asymmetrical factorial experiments. *J. R. S. S.* Series B, **10**, 109.
Nandi, H. K. (1947). A mathematical set up leading to analysis of a class of designs. *Sankhya*, **8**, 172.
Ramamurti, B. and B. Sitaraman (1942). On maximal sets of confounded interactions in $(2^n, 2^k)$ confounded design. *Sankhya*, **6**, 183.
Rao, C. R. (1945). Finite geometries and certain derived results in theory of numbers. *Proc. Nat. Inst. Sci.* (India), **2**, 136.
——— (1946*a*). Difference sets and combinatorial arrangements derivable from finite geometries. *Ibid.*, **3**, 123.
——— (1946*b*). Hypercubes of strength 'd' leading to confounded designs in factorial experiments. *Bull. Cal. Math. Soc.*, **38**, 67.
——— (1946*c*). Confounded factorial designs in quasi-Latin squares. *Sankhya*, **7**, 295.
——— (1947*a*). General methods of analysis for incomplete block designs. *J. Am. Stat. Assoc.*, **42**, 541.
——— (1947*b*). Factorial experiments derivable from combinatorial arrangements of arrays. *J. R. S. S. Suppl.*, **9**, 128.
——— (1950). The theory of fractional replication in factorial experiments. *Sankhya*, **10**, 81.
Saxena, P. N. (1950). A simplified method of enumerating Latin squares by Mac Mohan's differential operators. Part I. The 6×6 Latin square. *J. Ind. Soc. Agri. Stat.*, **2**, 106.
Shrikande, S. S. (1950). The impossibility of certain symmetrical balanced incomplete block designs. *Ann. Math. Stat.*, **21**, 106.

5. *Multivariate analysis*

Bhattacharya, A. (1946). On a measure of divergence between two multinormal populations. *Sankhya*, **7**, 401.
Bose, R. C. (1936). On the exact distribution and moment coefficients of the D^2 statistic. *Sankhya*, **2**, 143.
Bose, R. C. and Roy, S. N. (1938). The distribution of the Studentised D^2 statistic. *Sankhya* **4**, 19.
Mahalanobis, P. C. (1930). On tests and measures of group divergence. *Jour. Asiatic Soc. Bengal*, **26**, 541.
——— (1936). On the generalised distance in statistics. *Proc. Nat. Inst. Sci. India*, **2**, 49.
——— (1937). *Sankhya*, **3**, 35.
Mahalanobis, P. C., Bose, R. C. and Roy, S. N. (1937). Normalisation of statistical variates and the use of rectangular coordinates in the theory of sampling distributions. *Sankhya*, **3**, 1.
Nanda, D. N. (1948*a*). Distributions of a roots of a determinantal equation. *Ann. Math. Stat.*, **19**, 47.
——— (1948*b*). Limiting distribution of a root of a determinantal equation. *Ann. Math. Stat.*, **19**, 340.
——— (1949). Distribution of the sum of roots of a determinantal equation under a condition. *Ann. Math. Stat.*, **20**.
Narian, R. D. (1948). A new approach to sampling distributions of the multivariate normal theory. *J. Ind. Soc. Agri. Stat.*, **1**, 59.
——— (1949). Some results on discriminant functions. *J. Ind. Soc. Agri. Stat.*, **2**, 49.
Rao, C. R. (1945). Information and accuracy attainable in the estimation of statistical parameters. *Bull. Cal. Math. Soc.*, **37**, 81.
——— (1947). Tests with discriminant functions in multivariate analysis. *Sankhya*, **7**, 407.
——— (1948*a*). Tests of significance in multivariate analysis. *Biom.*, **35**, 58.
——— (1948*b*). The utilization of multiple measurements in problems of biological classification. *J.R.S.S.* Series B, **10**, 159.
——— (1949). On some problems arising out of discrimination with multiple characters. *Sankhya*, **9**, 343.
——— (1950). A note on the distribution of $D^2_{p+q} - D^2_p$ and some computational aspects of D^2 statistic and discriminant function. *Sankhya*, **10**, 257.
Rao, C. R and Slater, P. (1949). Multivariate analysis applied to differences between neurotic groups. *British J. Psychology, Statistics Section*, **2**, 17.

Roy, S. N. (1939). p-statistics or some generalisations in analysis of variance appropriate to multivariate problems. *Sankhya*, **3**, 341.

——— (1942). The sampling distribution of p-statistics and certain allied statistics on the non-null hypothesis. *Sankhya*, **6**, 15.

——— (1945). The individual sampling distribution of the maximum, the minimum and any intermediate of the p-statistics on the null hypothesis. *Sankhya*, **7**, 133.

——— (1946). Multivariate analysis of variance : The sampling distribution of the numerically largest of the p-statistics on the non-null hypothesis. *Sankhya*, **8**, 15.

Yardi, M. R. (1946). A statistical approach to the problem of chronology of Shakespeare's plays. *Sankhya*, **7**, 263.

6. *Theory of distributions and problems of tabulation*

Banerji, S. K. (1936). The one tenth percent level of the ratio of variances. *Sankhya*, **2**, 425.

Bhaba, H. J. (1950). On the stochastic theory of continuous parametric systems and its application in electron cascades. *Proc. Roy. Soc.*, London, Ser. A, **202**, 301.

Bhargava, R. P. (1946). Test of significance for intra-class correlation when the family sizes are not equal. *Sankhya*, **7**, 435.

Bhattacharya, A. (1945). On some sets of sufficient conditions leading to the normal bivariate distribution. *Sankhya*, **6**, 399.

Bose, P. K. (1943). On the confluent hypergeometric series, *Sankhya*, **6**, 407.

——— (1947). On the recursion formulae, tables and Bessel function population associated with the distribution of classical D^2-statistic. *Sankhya*, **8**, 235.

——— (1948). On the construction of an unbiased and most powerful critical region based on the classical D^2-statistic. (D. Phil. thesis, Calcutta University).

——— (1949). Tables of significant levels of Studentised D^2-statistic. *Bull. Cal. Stat. Assoc.*, **2**, 131.

Bose, P. K. and Roy, S. N. (1941). On the reduction formulae for the incomplete probability integral of the Studentised D^2-statistic. *Science and Culture*, **5**, 773.

Bose, R. C. (1946). The patch number problem. *Science and Culture*, **12**, 199.

Chakravarthy, M. (1946). The moments and Semi-invariants of the mean square successive difference. *Bull. Cal. Math. Soc.*, **38**, 185.

——— (1947). On a special case of the distribution law of the mean square successive difference. *Bull. Cal. Math. Soc.*, **39**, 15.

Chandrasekar, C. and Francis, M. (1941). A method to get the significance limit of a type of test criteria. *Sankhya*, **5**, 165.

Cheriyan, K. C. (1941). A bivariate correlated Gamma type distribution function. *J. Ind. Math. Soc.*, **5**, 133.

——— (1945). Distributions of certain frequency constants in samples from non-normal populations. *Sankhya*, **7**, 159.

Gayen, A. K. (1949). The distribution of Student's t in random samples of any size drawn from non-normal universes. *Biom.*, **37**, 353.

——— (1950*a*). The distribution of variance ratio in random samples of any size drawn from non-normal universes. *Biom.*, **37**, 236.

——— (1950*b*). The significance of difference between the means of two non-normal samples. *Biom.*, **37**, 399.

George, Aleyamma (1945). On the accuracy of different approximations to the L distribution. *Sankhya*, **7**, 20.

Ghurye, S. G. (1949). Transformations of a binomial variate for the analysis of variance. *J. Ind. Soc. Agri. Stat.*, **2**, 94.

Ghosh, M. N. (1946). On the order of approximation in the Central limiting theorem of probability of Laplace and Liapounoff. *Sankhya*, **7**, 323.

——— (1947). Test of randomness. *Bull. Cal. Stat. Assoc.*, **1**, 135.

——— (1948). A test for field uniformity based on the space correlation method. *Sankhya*, **9**, 39.

Kibble, W. F. (1940). A two-variate Gamma type distribution. *Sankhya*, **5**, 137.

Krishna Iyer, P. V. (1945). The use of generalised Dirichlet's integral in solving some distribution problems of statistics. *Proc. Ind. Acad. Sci.*, **22**, 76.

——— (1948*a*). Random association of points on a lattice. *Nature*, **162**, 333.

——— (1948*b*). The theory of probability distribution of points on a line. *J. Ind. Soc. Agri. Stat.*, **1**, 173.

——— (1949*a*). A new theorem useful in calculating the factorial moments of certain probability distributions. *Nature*, **164**, 282.

——— (1949*b*). The first and second moments of some probability distributions arising from points on a lattice and their application. *Biom.*, **36**, 135.

——— (1950). Further contributions to the theory of probability distributions of points on a line. *J. Ind. Soc. Agri. Stat.*, **2**, 141.

Krishnan Nair, A. N. (1941). Distribution of Student's t and correlation coefficient in samples from non-normal population. *Sankhya*, **5**, 383.

Krishnasastry, K. V. (1946). On a certain distribution in the theory of sampling. *Proc. Nat. Inst. Sci.*, (India), **12**, 427.

Nair, K. R. (1939*a*). The median in tests by randomisation. *Sankhya*, **4**, 543.
——— (1939*b*). Table of confidence interval for the median in samples from any continuous population. *Sankhya*, **4**, 551.
——— (1947). A note on the mean deviation from the median. *Biom.*, **34**, 360.
——— (1948). The distribution of the extreme deviate from the sample mean and its Studentized form. *Biom.*, **35**, 118.
Nair, U. S. (1939). The application of moment functions in the study of distribution laws in statistics. *Biom.*, **30**, 274.
Panse, V. G. and Aiycht, G. R. (1944). Tables of 10 percent points of z and the variance ratio. *Ind. J. Agri. Sci.*, **14**, 244.
Patnaik, P. B. (1949). The non-central χ^2 and F distributions and their applications. *Biom.*, **36**, 202.
——— (1950)). The use of mean range as an estimator of variance in statistical tests. *Biom.*, **37**, 78.
Pearson, E. S. and Adyanthayya, N. K. (1929). The distribution of frequency constants in small samples from non-normal symmetrical and skew populations. *Biom.* **21**, 259.
Pillai, K. C. S. (1948). A note on ordered samples. *Sankhya*, **8**, 375.
——— (1950). On the distribution of mid-range and semi-range in samples from a normal population. *Ann. Math. Stat.*, **21**, 100.
Rao, C. R. (1942). On bivariate correlation surfaces. *Science & Culture*, **8**, 236.
——— (1948). Large sample tests of statistical hypotheses concerning several parameters with applications to problems of estimation. *Proc. Camb. Phil. Soc.*, **44**, 40.
Rao, K. S. and Kendall, D. G. (1950). On the generalised second limit theorem in the calculus of probabilities. *Biom.*, **37**, 224.
Srivastava, M. P. (1941). On bivariate correlation surfaces. *Science & Culture*, **6**, 615.
Sukhatme, B. V. (1949). Random association of points on a line. *J. Ind. Soc. Agri. Stat.*, **2**, 60.
Sukhatme, P. V. (1936*a*). On the analysis of k samples from exponential population with special reference to the problem of random intervals. *Stat. Res. Memoirs.*, **1**, 94.
——— (1936*b*). A contribution to the problem of two samples. *Proc. Ind. Acad. Sci.*, A, **2**, 584.
——— (1937*a*). The problem of k samples for Poisson population. *Proc. Nat. Inst. Sci. India*, **3**, 297.
——— (1937*b*). Tests of significance for samples of the χ^2 population with two degrees of freedom. *Ann. Eugen.*, **8**, 52.
——— (1938*a*). On Fisher and Behren's test of significance for the difference in means of two normal samples. *Sankhya*, **4**, 39.
——— (1938*b*). On Fisher's combinatorial methods giving moments and cumulants of the distribution of k-statistics. *Sankhya*, **4**, 53.
——— (1939). On bi-partitional functions. *Philos. Trans Roy. Soc.*, Series A, No. 780.
——— (1943). Moments and product moments of moment statistics for samples of the finite and infinite population. *Sankhya*, **6**, 363.
Swarup, S. (1938). Tables of exact values of probabilities for testing the significance of difference between proportions based on pairs of small samples. *Sankhya*, **4**, 73.
Vaswani, S. P. (1947). A pitfall in correlation theory. *Nature*, **160**, 405.
Vaswani, Sundri (1950). Assumptions underlying the use of the tetrachoric correlation coefficient. *Sankhya*, **10**, 269.
Uttam Chand (1950). Distributions related to comparisons of two means and two regression coefficients. *Ann. Math. Stat.*, **21**, 507.

7. *Estimation*

Bhattacharya, A. (1946, 1947, 1948). On some analogues of the amount of information and their use in statistical estimation. *Sankhya*, 8, 1, 201, 315.
——— (1950). Unbiased statistics with minimum variance. *Proc. Roy. Soc. Edin.*, A. **63**, 69.
Bose, R. C. (1944). The fundamental theorem of linear estimation. *Proc. Ind. Sci. Congress*, Part III, 4.
George, Aleyamma (1942). On the problem of interval estimation. *Sankhya*, **6**, 111.
Ghurye, S. G. (1950). A method of estimating the parameters of an autoregressive time series. *Biom.*, **37**, 173.
Huzurbazar, V. (1948). The likelihood equation, consistency and the maximum of the likelihood function. *Ann. Eugen.*, **14**, 185.
——— (1949). On a property of distributions admitting sufficient statistics. *Biom.*, **36**, 71.
Kosambi, D. D. (1947). An extension of least square method in statistical estimation. *Ann. Eugen.*, **13**, 257.
Pillai, K. C. S. (1946). Confidence interval for the correlation coefficient. *Sankhya*, **7**, 415.
Nair, K. R. (1950). Efficiencies of certain linear systematic statistics for estimating dispersion from normal samples. *Biom.*, **37**, 182.
Rao, C. R. (1945*a*). Generalisation of Markoff's theorem and tests of linear hypotheses. *Sankhya*, **7**, 9.

——— (1945*b*). Markoff's theorem with linear restrictions on parameters. *Sankhya*, **7**, 16.
——— (1945*c*). Information and accuracy attainable in the estimation of statistical parameters. *Bull. Cal. Math. Soc.*, **37**, 81.
——— (1946*a*). On the linear combination of observations and the general theory of least squares. *Sankhya*, **7**, 237.
——— (1946*b*). Minimum variance and the estimation of several parameters. *Proc. Camb. Phil. Soc.*, **43**, 280.
——— (1948). Sufficient statistics and minimum variance estimates. *Ibid.*, **45**, 215.
Seth, G. R. (1949). On variance of estimates. *Ann. Math. Stat.*, **20**, 1.

8. *Testing of hypothesis*

Bahadur, Raghu Raj (1950). On a problem in the theory of k populations. *Ann. Math. Stat.*, **21**, 362.
Bahadur, Raghu Raj and Robbins, H. (1950). The problem of the greater mean. *Ann. Math. Stat.*, **21**, 469.
Divatia, V. V. (1949). A note on sequential exponential tests. *J. Ind. Soc. Agri. Stat.*, **2**, 86.
Ghosh, M. N. (1948). On the problem of similar regions. *Sankhya*, **8**, 329.
Nair, U. S. (1941). A comparison of tests for the significance of the difference between two variances. *Sankhya*, **5**, 157.
Nandi, H. K. (1946*a*). On the average power of test criteria. *Sankhya*, **8**, 67.
——— (1946*b*). On the power function of studentised D^2-statistic. *Bull. Cal. Math. Soc.*, **38**, 39.
——— (1947). A note on Student's t for paired samples. *Bull. Cal. Math. Soc.*, **39**, 61.
——— (1948). Use of well known statistics in sequential analysis. *Sankhya*, **8**, 339.
Narain, R. D. (1949). On the completely unbiased character of tests of independence in multivariate normal systems. *Current Science*, **18**, 41.
Patnaik, P. B. (1948). The power function of the test for the difference between proportions in a 2×2 table. *Biom.*, **35**, 157.
Poti, S. J. (1950). Power function of χ^2 test with special reference to analysis of blood group data. *Sankhya*, **10**, 397.
Rao, C. R. (1950*a*). Statistical inference applied to classificatory problems. Part I. *Sankhya*, **10**, 229.
——— (1950*b*). Sequential tests of null hypotheses. *Sankhya*, **10**, 361.
Roy, S. N. (1947). Notes on testing of composite hypothesis. *Sankhya*, **8**, 257.
——— (1948). Notes on testing of composite hypothesis II. *Sankhya*, **9**.
——— (1950). Univariate and multivariate analysis as problems in testing of composite hypotheses. *Sankhya*, **10**, 29.

9. *Anthropometry*

Chattopadhyay, K. P. (1942). Application of statistical methods to anthropological research. *Sankhya*, **6**, 99.
——— (1949). Comparability of measurements. *Sankhya*, **9**, 399.
Karve, I. and Dandekar, V. M. (1951). Anthropometric measurements of Maharashtra. Deccan College monograph series : No. 8, Poona.
Mahalanobis, P. C. (1927). Analysis of race mixture in Bengal. *J. Asiatic Soc.* (Bengal), **23**, 301.
——— (1930). A statistical study of certain anthropometric measurements from Sweden. *Biom.*, **22**, 94.
——— (1937). On the accuracy of profile measurements with a photographic profiloscope. *Sankhya*, **3**, 65.
Mahalanobis, P. C., Majumdar, D. N. and Rao, C. R. (1949). Anthropometric survey of the United Provinces, 1941. A Statistical study. *Sankhya*, **9**, 90.
Majumdar, D. N. (1940). Need of standardisation of anthropometric technique. *Sankhya*, **5**, 591.
——— (1948). Maha Gujarat racial and serological survey, Part I, *J. Gujarat Res. Soc.*, **9**, 1.

10. *Population studies and vital statistics.*

Chandrasekar, C. (1949*a*). Some aspects of Parsi demography. *Human biology*, **20**, 77.
——— (1949*b*). The effect of the change in mortality conditions in an age group on the expectation of life at birth. *Ibid.*, **21**, 35.
Chandrasekar, C. and Sen, M. (1948). Enquiry into the reproductive patterns of urban and rural populations. Report of the scientific advisory board for 1948, p. 173.
Chandrasekar, C. and Deming, E. (1949). On a method of estimating birth and death rates and extent of registration. *J. Am. Stat. Assoc.*, **44**, 101.
Mathen, K. K. and Lal, R. B. (1946). Studies in the health problems of a rural community in Bengal. *Indian Medical Gazette*, **81**, 203.
Sovani, N. V. (1948). The social survey of Kolhapur City, Part I—Population and fertility. Publication No. 18, Gokhale Institute of Politics and Economics.

11. *Economic Statistics*

Bose, S. R. (1937). An index of business activity. *Ind. J. Econ.*, 25.
'Capital', (1938). March 17, 392.
Desai, R. C. (1948). Consumer expenditure in India 1931-32 to 1940-41. *J.R.S.S.*, **111**, 261.
Gadgil, D. R. (1948). Report of a survey of the direct and indirect benefits of the Godavari and Pravara canals. Publication No. 17, Gokhale Institute of Politics and Economics.
Gadgil, D. R. and Gadgil, V. R. (1933). A survey of the marketing of fruit in Poona. Publication No. 3. Gokhale Institute of Politics and Economics.
——— (1940). A survey of farm business in Wai Taluka. Publication No. 7, *ibid.*
Gadgil, D. R. and Gogate, L. V. (1936). A survey of motor bus transportation in six districts of Bombay Presidency. Publication No. 4. *Ibid.*
Ghosh, Ambica (1950*a*). Economic classification of agricultural regions in Bengal, *Sankhya*, **10**, 109.
——— (1950*b*). A note on the marginal and optimum size of holding in Bengal. *Sankhya*, **10**, 383.
Kakade, R. G. (1947). The socio-economic survey of weaving communities in Solapur. Publication No. 14, Gokhale Institute of Politics and Economics.
Madhava, K. B. (1941). Trade Unionism. *Ind. J. Econ.*, 8.
Madhava, K. B. and Krishna Sastry, T. (1941). A study of trade unionism in India, *Sankhya*, **5**, 209.
Mahalanobis, P. C. (1941). A statistical report on the rupee census. *Report on currency and finance*, 1940-41, 49.
Meek, D. (1937). Some measure of industrial activity in India. *J.R.S.S.*, **100**, 364.
Murti, V. N. (1947*a*). The average life of one rupee notes. *Reserve Bank of India Bulletin*, 549.
——— (1947*b*). Classification of Banks. *Reserve Bank of India Bulletin*, 355.
Poti, S. J. (1946). A study of expenditure pattern of Calcutta middle class family budgets. *Sankhya*, **7**, 425.
Sastry, N. S. R. (1941). Industrial productive activity in India 1932-38, *Sankhya*, **5**, 191.
Sastry, N. S. R. and Mathew, N. T. (1940). Trends of foreign trade of India during the past sixty years. *Sankhya*, **4**, 479.
Sinha, A. R. (1939). Inter-relation between supply and price of raw jute. *Sankhya* **4**, 397.
——— (1940). A preliminary study of the distribution of expenditure of working class families in Bombay and Madras Cities. *Sankhya*, **4**, 461.
Sinha, H., & Roy, P. R. (1941). Index of business activity in India, *Sankhya*, **5**, 179.

12. *Education and Psychology*

Hossain, Q. M. (1940). A note on examination marks. *Sankhya*, **4**, 563.
——— (1941). Standardisation of examination marks. *Sankhya*, **5**, 295.
Krishnaswamy Ayyanger (1941). Random lines—A statistical study. *Sankhya*, **5**, 307.
Kuppuswamy, B. (1941). Are habits transmitted? *Sankhya*, **5**, 301.
Mahalanobis, P. C. (1933). Studies in educational tests. No. 1. The reliability of a group test of intelligence in Bengali. *Sankhya*, **1**, 25.
——— (1934). Studies in educational tests. No. 2. The age variation of scores in a group test of intelligence in Bengali. *Sankhya*, **1**, 231.
Sen, Amiya (1950). A statistical study of Rorschah test. *British J. Psychology* (*stat.*), **3**, 21.

13. *Genetics*

Bhat, N. R. (1948). An improved genetical map of Punnett's B-Chromosome in sweet pea, *Lythrus odoratus.*, *J. Genet.*, **48**, 343.
Kosambi, D. D. (1944). The estimation of map distance from recombination values. *Ann. Eugen.*, **12**, 172.
Nanda, D. N. (1949*a*). The standard errors of discriminant function coefficients in plant selection. *J.R.S.S.* (Series B), **11**, 283.
——— (1949*b*). Efficiency of the application of discriminant function in plant selection. *J. Ind. Soc. Agri. Stat.*, **2**, 8.
Panse, V. G. (1940). Inheritance of quantitative characters and plant breeding. *J. Genet.*, **40**, 283.
——— (1947). Symposium on statistical methods in plant and animal breeding. *Proc. Ind. Acad. Sci.*, **25**, 126.
Panse, V. G. & Bokil, S. D. (1948). Estimation of genetic variability in plants. *J. Ind. Soc. Agri. Stat.*, **1**, 80.
Rao, C. R. (1950). Methods of scoring linkage data giving the simultaneous segregation of this factors. *Heredity*, **4**, 37.
Sukhatme, P. V. (1941). On Bernstein's improved method of estimating the blood-group gene frequencies. *Sankhya*, **6**, 85.

14. *Quality control*

Bose, Chameli (1949). Application of statistical quality control technique, Bulletin No. 1, Indian Society for Quality Control.

Nair, K. R. (1949). On the application of statistical quality control methods in wood based industries. *Ind. Forest. Leaflet*, No. 109.

Pran nath (1948). Percentage inspection *J.R.S.S.* Series A, **111**, 235.

Sundri, Vaswani (1949). Some advantages of introducing statistical investigation in textile mills. *ATIRA Research Notes*, **1**, No. 1.

——— (1950). Quality control in blowing room and its effect on production in subsequent departments. *ATIRA Research Notes*, **1**, No. 2.

15. *Meteorology*

Banerji, S. K. (1950). Methods of foreshadowing monsoon and winter rainfall in India. *Ind. J. Met. and Geophysics*, **1**, 1.

Normand, C. W. B. (1932). Some problems of modern meteorology, No. 6. The present position of seasonal weather forecasting. *Q. J. Meteor. Soc.*, **58**, 3.

Savur, S. R. (1932). On the 'performance test' in statistics. *Ind. J. Phy.*, **7**, 27.

Walker, G. T. (1914). *Mem. Ind. Met. Dept.*, **21**, 13.

16. *Miscellaneous*

Koshal, R. S. (1933). Application of the method of maximum likelihood to the improvement of curves fitted by the method of moments. *J.R.S.S.*, **96**, 303.

——— (1935). Application of the method of maximum likelihood to the derivation of efficient statistics for fitting frequency curves. *J.R.S.S.*, **98**, 128.

Koshal, R. S. and Turner, A. J. (1930). Studies in the sampling distribution of cotton for the determination of fibre properties. *J. Text. Inst.*, **21**, T 325-T 370.

Kosambi, D. D. (1942). On valid tests of linguistic hypothesis. *New Indian Antiquity*, **5**, 21.

Mahalanobis, P. C. (1937). A statistical note on the Hooghly-Howrah flushing and irrigation scheme. Published by the Bengal Government.

Rao, C. R. & Shaw (1948). On a formula for the prediction of cranial capacity. *Biometrics*, **4**, 247.

Singh, J. (1946). Theories of probability. *Sankhya*, **7**, 257.

PUBLICATIONS OF THE NATIONAL INSTITUTE OF SCIENCES OF INDIA

PROGRESS OF SCIENCES IN INDIA

No. 1, 1939—1950. Physics—including Meteorology, Geophysics and Solar Physics, (pp. 1-80). Rs.3.12 nP.

No. 2, 1939—1950. Chemistry—including Physical, Inorganic, Analytical and Organic Chemistry. (pp. 1-97). Rs.3.75 nP.

No. 3, 1938—1950. Zoology—including General Zoology, Entomology, Fish and Fisheries, Helminthology and Parasitology. (pp. i-v and 1-189). Rs.7.50 nP.

No. 4, 1938—1950. Geology—including Stratigraphy, Mineralogy and Petrology, Geography and Palaeontology and Palaeobotany. (pp. 1-138). Rs.5.75 nP.

No. 5, 1939—1950. Botany—including Algology, Mycology and Plant Pathology, Bryology, Pteridology, Gymnosperms, Angiosperms and Applied Botany. (pp. i-iii & 1-263). Rs.10.62 nP.

Published by the National Institute of Sciences of India, Mathura Road, New Delhi, and Printed by S. N. Banerjea, The Lotus Press, 52/9, Bipin Behary Ganguly Street, Calcutta-12.